

Vectors

Definitions

Vector - value with magnitude & direction

Must have arrows when drawn

Scalar - value with magnitude only

Resultant - The sum of two or more vectors

Equilibrant - equal and opposite in magnitude to resultant

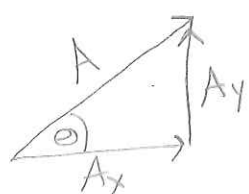
Magnitude - size of a vector value

Useful Equations

$$A_x = A \cos \theta \quad A = \frac{A_x}{\cos \theta} \quad \theta = \cos^{-1} \frac{A_x}{A}$$

$$A_y = A \sin \theta \quad A = \frac{A_y}{\sin \theta} \quad \theta = \sin^{-1} \frac{A_y}{A}$$

$$\theta = \tan^{-1} \frac{A_y}{A_x} \quad A = \sqrt{A_x^2 + A_y^2}$$



A_x = horizontal

A_y = vertical

A = hypotenuse

θ = between A_x & A

Examples of Vectors

displacement, velocity, acceleration

Force, momentum, impulse

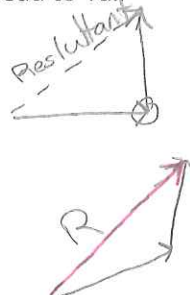
Field strength

Examples of Scalars

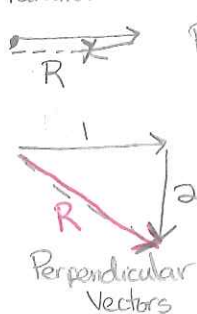
distance, speed, mass, time,
energy, work, power, current,
potential difference, wavelength,
frequency, charge

"Graphical" Addition of Vectors

Head to Tail



Parallel



Resultant from
start of first
to end of
last

Tail to tail (Parallelogram)



Determining the Maximum or Minimum Resultant

The max resultant occurs when the difference between vectors is 0°
(or the smallest angle) which means same direction

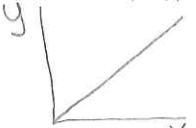
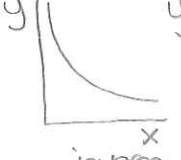
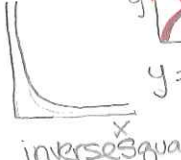


The min resultant occurs when the difference between vectors is 180°
(or largest angle up to 180°) which means opposite direction



Graph Related Skills

Finding Slope	Significance of Slope	Area under the graph
$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$ $m = \frac{\text{change in "y" variable}}{\text{change in "x" variable}}$	Match the graph expressed as $y/x = m$ to an equation from ref. table. Ex: $\frac{\text{Equation}}{f} = h$ $\frac{d}{t} = v$ $a = \frac{\Delta v}{t}$ $\frac{F_g}{m} = g$ $\frac{V}{\lambda} = f$ $R = \frac{V}{I}$ $V = \frac{W}{q}$ $\frac{F_{\text{net}}}{a} = m$	Multiply "y" "x" and match to equation from ref. table If shape of graph is rectangle use $A = y \cdot x$ If shape of graph is triangle use $A = \frac{1}{2} y \cdot x$ Ex: $PE_s = \frac{1}{2} F_s x$ $W = Fd$ $W = \frac{1}{2} Fd$ $v = f \lambda$ constant Force For avg force

Graphical Relationships (Relationship types)				
 no relationship "y" is not influenced by a change in "x"	 direct relationship = whatever happens to "x", happens to "y" Ex: If "x" doubles "y" doubles Equation $y = mx$	 direct square - whatever happens to "x", happens squared to "y" Ex: "x" times 3 means "y" times 9 Equation $y = mx^2$	 inverse - whatever happens to "x", the opp happens to "y" Equation $y = \frac{m}{x}$	 inverse square - whatever happens to "x", the opp square happens to "y" Equation $y = \frac{m}{x^2}$

Rules for graphing and interpreting data

Scaling the axes

make sure all increments are equal for the same axis

Best Fit Line

use to average data
Leave some points above and some below

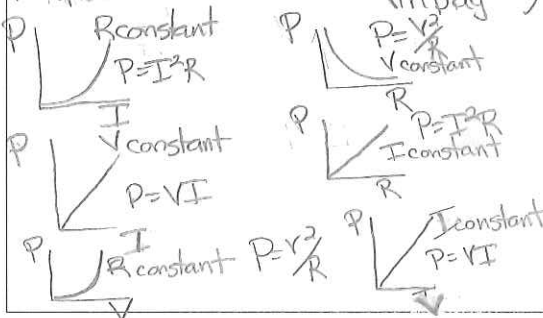
Slope from Best Fit

Pick points from best fit not plotted points
equation
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Use caution with graphs of the following concepts or equations

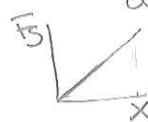
Electrical Power

Determine which variable is dependent independent or constant. (3rd variable in play)



Hooke's Law and Spring Elastic Energy

For Springs the Force on the spring is not constant → it starts at 0 and increases at a constant rate so $\frac{F_s}{2}$



$$W = \frac{1}{2} F_s x = PE_s$$

$$PE_s = \frac{1}{2} k x^2$$

$$W_{\text{spring}} = PE_s$$

Assigning Units, Estimating Values and Rearranging Equations

Base "SI" units

Mass = kilograms

Length or position - meters

Time - seconds

Charge - coulombs

Derived Units

Force - Newtons = kg m/s^2

Work/Energy Joules = $\text{kg m}^2/\text{s}^2$

$W = PE = KE = PE_s$
 $Fd = mgh = \frac{1}{2}mv^2 = \frac{1}{2}kx^2 = VC$

Power = Watts = $\text{J/s} = \frac{\text{Nm}}{\text{s}}$

$P = W/t$

Current - Amperes = C/s

$I = Q/t$

Potential Difference - $V = \text{J/C}$

$V = W/Q$

Resistance = Ohms

$R = V/I$

Frequency = Hertz = $1/\text{s}$

$f = 1/T$

Deriving alternate units from an equation

$$v = f\lambda = (\text{Hz})(\text{m}) = \text{m/s} \quad I = \frac{Q}{t} = \left(\frac{\text{C}}{\text{s}}\right)$$

$$v = d/t = \left(\frac{\text{m}}{\text{s}}\right)$$

$$W = (Vq = eV)$$

$$F = ma = (\text{kg m/s}^2)$$

$$P = I^2 R = (\text{A}^2 \Omega)$$

$$p = mv = (\text{kg m/s})$$

$$V = \frac{W}{q} = \left(\frac{\text{J}}{\text{C}}\right)$$

$$J = F_{\text{net}} t = (\text{Ns} = \text{kg m/s}^2 \cdot \text{s})$$

$$P = VI = \left(\frac{\text{J}}{\text{C}} \cdot \frac{\text{C}}{\text{s}} = \frac{\text{J}}{\text{s}}\right)$$

$$k = \frac{F_s}{x} = (\text{N/m})$$

$$P = VI = \left(\frac{\text{J}}{\text{C}} \cdot \frac{\text{C}}{\text{s}} = \frac{\text{J}}{\text{s}}\right)$$

$$W = Fd = (\text{Nm})$$

$$v = f\lambda = (\text{Hz} \cdot \text{m} = \text{m/s})$$

$$PE = mgh = (\text{kg m/s}^2 \cdot \text{m})$$

$$v = \lambda/T = (\text{m/s})$$

$$P = W/t = \left(\frac{\text{J}}{\text{s}}\right)$$

$$E_{\text{photon}} = hf = (\text{Js} \cdot \text{Hz} = \frac{\text{Js}}{\text{s}} = \text{J})$$

$$P = VI = (\text{VA})$$

$$E = \frac{F_e}{q} = \left(\frac{\text{N}}{\text{C}}\right)$$

DO NOT MEMORIZE - Practice Skill

Scientific Notation and the Metric system

$$420 \text{ nm} = 4.2 \times 10^2 \times 10^{-9} \text{ m} = 4.2 \times 10^{-7} \text{ m}$$

$$1.6 \times 10^{-5} \text{ A} = 1.6 \times 10^1 \times 10^{-6} \text{ A} = 16 \mu\text{A}$$

$$1.6 \times 10^{-4} \times 10^{-3} \text{ A} = .016 \text{ mA}$$

$$6.4 \times 10^{-7} \text{ m} = 6.4 \times 10^2 \times 10^{-9} \text{ m} = 640 \text{ nm}$$

Rearranging Equations

Determine which variable the equation should be written "in terms of"

- isolate by multiplying, dividing etc

Ex = solve for t in $d = v_i t + \frac{1}{2} a t^2$
 when d & $a = g$ & $v_i = 0$

$$t = \sqrt{\frac{2d}{g}}$$

solve for t when $v = d/t$ $t = d/v$

How to make a "triangle"

Start from equation in format of $y = mx$



$$Ex = d = vt$$



$$F_{\text{net}} = ma$$



Estimating Values for mass, weight, height/length, time

Put all values in terms of the most reasonable metric unit and eliminate #'s that don't make sense.

Keep in mind the following metric-English equivalents

$$1 \text{ kg} = 2.2 \text{ lbs}$$

$$1 \text{ ft} = 30 \text{ cm}$$

Use metric prefix chart on PPT

$$1 \text{ kg} = 1 \times 10^3 \text{ g} = 1000 \text{ g}$$

$$1 \text{ mm} = 1 \times 10^{-3} \text{ m} = .001 \text{ m}$$

$$25 \text{ cm} = 25 \times 10^{-2} \text{ m} = 2.5 \times 10^{-1} \text{ m}$$

$$3.5 \times 10^{-1} \mu\text{m} = 3.5 \times 10^{-5} \text{ m}$$

KINEMATICS GRAPHS

Vocabulary:

Distance

Scalar - length without direction
- change in position w/o direction

Displacement

Vector - length with direction
- change in position with direction

Velocity - rate of change in position
with direction $v = \frac{d}{t}$

Speed - rate of change in position
without direction

Acceleration - rate of change in
velocity
change in speed or direction

Equilibrium - $F_{net} = 0$ $a = 0$
not speeding up or slowing down

Net Force - The unbalanced force
"The amount a force wins by"

Constant unbalanced force
Constant $F_{net} = \text{constant acc}$

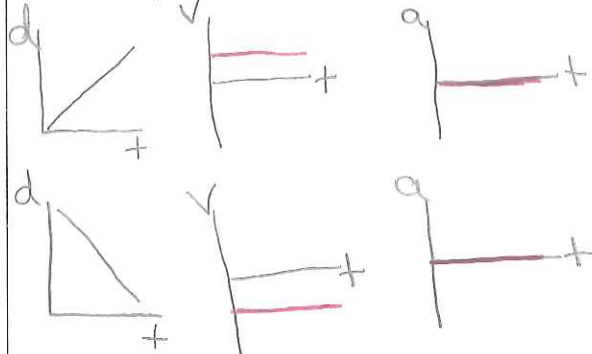
Objects at rest (Equilibrium option 1)



object at rest, stays at rest

No Net force, No "a"

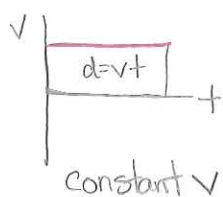
Constant speed or velocity (Equilibrium option 2)



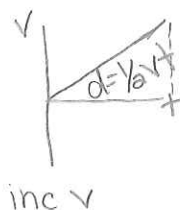
No net force - object in motion
remains in motion - no "a"

- NOT SPEEDING UP or SLOWING DOWN

Distance/displacement from a v vs. t graph



d is the area of
 v vs t
 $d = vt$

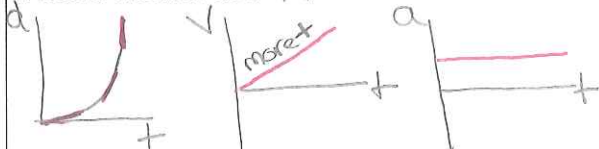


$$\bar{v} = \frac{v_i + v_f}{2}$$

$$d = \bar{v}t$$

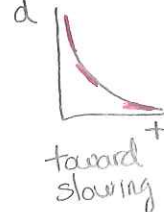
Accelerated motion

acceleration & F_{net} do the same thing



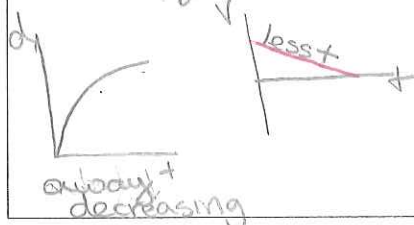
away +
increasing

constant +
acceleration
means speeding up



toward
slowing

constant
acc



away +
decreasing



toward
speeding up

constant
acc

Accelerated Motion

Acceleration
Change in velocity
speed or direction $a = \frac{\Delta v}{t}$
 $a_c = \frac{v^2}{r}$

Average speed/velocity
 $\bar{v} = \frac{v_i + v_f}{2}$

Initial Velocity
 v_i

Final Velocity
 v_f

Distance/Displacement
 d

Equilibrium
 $F_{net} = 0$ $a = 0$

Non-Equilibrium
 F_{net} is present
 F_{net} constant; a is constant

Net Force
 $F_{net} = ma$

Centripetal Acceleration
 $a_c = \frac{v^2}{r}$

Kinematics equations:

When acceleration equals zero use the equation

$v = d/t$ $d = vt$ $t = d/v$

When acceleration (net force) is present use these equations:

$v_f = v_i + at$ $v_f = at$ $\Delta v = at$
 $v_f^2 = v_i^2 + 2ad$ if $v_i = 0$ $v_f = \sqrt{2ad}$
 $d = v_i t + \frac{1}{2}at^2$ $d = \frac{1}{2}at^2$



OR if possible find average v using equation

$$\bar{v} = \frac{v_i + v_f}{2}$$

And use the equation

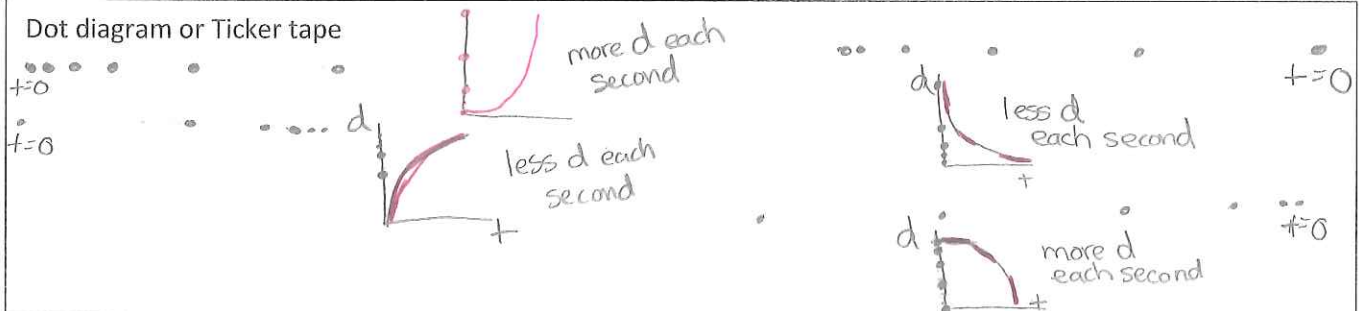
$$d = \bar{v}t$$

Acceleration can also be found using Newton's 2nd Law

$F_{net} = ma$ $a = \frac{F_{net}}{m}$

$$F_{net} = m \frac{\Delta v}{t}$$

Dot diagram or Ticker tape



Short cuts for $v_i = 0$

$d = \frac{1}{2}at^2$ $t = \sqrt{\frac{2d}{a}}$
 $v_f = at$ $v_f = \sqrt{2ad}$

"Head Problem" Method

v_i	v_f	$\bar{v}$	d	a	t
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Free Fall and other Projectiles

Vocabulary

Acceleration due to gravity

$$9.8 \text{ m/s}^2 \text{ Earth} \quad \left\{ \begin{array}{l} g = \frac{F_g}{m} \\ g = 9.8 \text{ m/s}^2 \end{array} \right.$$

Gravitational Field Strength

$$9.8 \text{ N/kg}$$

Force due to gravity

- Weight

Free Fall

- $a = g$ no air resistance

Projectile

The only force is gravity F_g

Horizontal Component

remains constant

Vertical Component

V_{iy} equal but opp to V_{fy}
 $V_y = 0$ at top

Initial Horizontal Velocity

$$V_{ix} = V_i \cos \theta$$

Initial Vertical Velocity

$$V_{iy} = V_i \sin \theta$$

Vertical Acceleration

$$a_y = g = 9.8 \text{ m/s}^2 \text{ downward}$$

Range

horizontal displacement
 $d_x = v_x t$ + determined using vertical axis

Launch Angle

$$\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right) \quad 45^\circ \text{ max range}$$

Max height/high pt

$$V_{iy} = 0 \quad \frac{1}{2} \text{ total time}$$

Rules for all projectiles

The net force is Weight- F_g so $a = g = 9.81 \text{ m/s}^2$ downward (vertical axis only)

$$a_x = 0$$

$$a_y = g$$

x at equilibrium

y experiencing net force

Objects dropped from rest ($v_{iy} = 0$) (downward can be assigned +)

$F_{\text{net}} = F_g = mg$ $g = 9.8 \text{ m/s}^2$ $d = \frac{1}{2} a t^2$ $v_f = g t$

If down is (downward)

v_i	a	t	d	a	t
0	10 m/s^2	5 s	5 m	10 m/s^2	1 s
0	20 m/s^2	10 s	20 m	10 m/s^2	2 s
0	30 m/s^2	15 s	45 m	10 m/s^2	3 s
0	40 m/s^2	20 s	80 m	10 m/s^2	4 s
0	50 m/s^2	25 s	125 m	10 m/s^2	5 s

For 1s to .5s change to cm

BENCHMARKS USING 10 m/s^2

Objects launched upward (down must be assigned -)

" V_i " and " a " are different directions so sign required

time up = time down

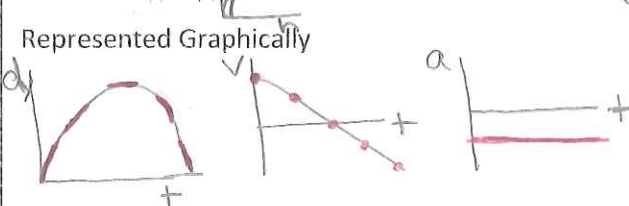
V_{iy} equal and opposite V_{fy}

V_y at top = 0

total vertical d ($d_i = 0$)

$$V_{\text{or } V_i} \quad V_f = \sqrt{2gh}$$

Represented Graphically



$$a_y = -9.81 \text{ m/s}^2$$

$$V_f = V_i + at \quad 0 = v_i + at$$

$$t = \frac{\Delta v}{a}$$

$$\text{total time}$$

$$t = \frac{2V_{iy}}{a}$$

$$d = \frac{1}{2} a t^2 \text{ (use falling side)}$$

up = down

Horizontal Projectiles

y axis

$F_{\text{net}} = F_g = mg$

horizontal

v_x d_x t

time is the same for both axes

$$d = \frac{1}{2} a t^2 \text{ if } d(x) \text{ and } d(y)$$

$$t = \sqrt{\frac{2d}{a}} \text{ if } d(x) \text{ and } d(y)$$

$$d(x) = \frac{1}{2} a t^2$$

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Projectiles at an angle.

Combine x axis at equilibrium

($v = d/t$)

with object launched upward

need to break v_i into horizontal & vertical components

$$V_{ix} = V_i \cos \theta$$

$$V_{iy} = V_i \sin \theta$$

$$\text{max range} = d_x = 45^\circ$$

$$\text{(angle closest to } 45^\circ)$$

$$\text{- equal difference from } 45^\circ$$

$$\text{is same landing spot}$$

$$\text{- max height equals}$$

$$\text{greatest time in air}$$

$$\text{highest angle}$$

$$V_x \text{ constant } V_y \text{ changes}$$

Newton's Laws and Dynamics

First Law

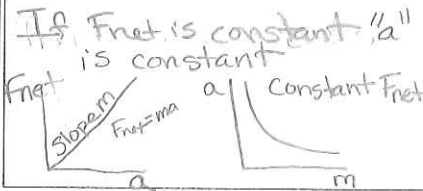
Law of Inertia
(Inertia is mass)
at rest, stay at rest
in motion, stay in motion
unless an unbalance (F_{net})
is present.
ie Velocity (speed or direction) only
changes if a net force
is present

Second Law

$$F_{net} = ma$$



Whatever F_{net} does
acceleration does



Third Law

For every force there exists
an equal and opposite
force
(Newton's 3rd Law force pairs)
Pairs act on each object

Vocabulary

Inertia — mass

Equilibrium = $F_{net} = 0$
 $a = 0$

Net Force $\rightarrow$ Cause acceleration
Unbalanced Force $\rightarrow$ Speed up
Slow down
change direction

Applied Force F_A

Horizontal Component of Force

$$F_{Ax} = F_A \cos \theta$$

Vertical Component of Force

$$F_{Ay} = F_A \sin \theta$$

Parallel component of weight

$$F_{g\parallel} = F_g \sin \theta = mg \sin \theta$$

Perpendicular component of weight

$$F_{g\perp} = F_N = mg \cos \theta$$

Resultant

Sum of 2 or more vectors

Equilibrant

equal but opposite of Resultant

Action Reaction Pair

Equal but opp force

Weight

$$F_g = mg$$

F_{net} on a projectile

Normal Force

$$F_N = F_g \text{ level surface}$$

$F_N = F_g \sin \theta$ on incline

Frictional Force

$$F_f = \mu F_N$$

opposes motion

When $F_{net} = 0$

Equilibrium

Set Forces from an
axis equal to zero
if v is 0 or constant

$$\text{If } F_{net} = 0 \quad a = 0$$

object remains at rest or
constant velocity

Friction scenario

$$F_{net} = F_A + F_f \quad 0 = F_A + F_f \quad F_A = -F_f$$

Lift scenario

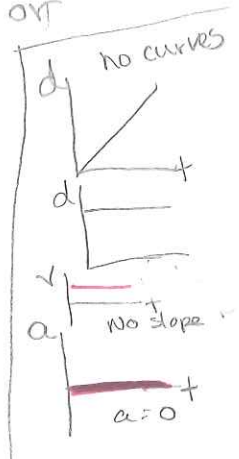
$$F_{net} = F_A + F_g \quad 0 = F_A + F_g \quad F_A = -F_g$$

Other Examples

$$F_{net \ x} = F_{Ax} + F_{fx}$$

$$F_{net \ y} = F_{gy} + F_{Ay}$$

$$F_{net \ \perp} = F_{g\perp} + F_N$$



When F_{net} is present (always constant)

- If F_{net} is present acceleration is present

- Whatever F_{net} does acceleration does

- Set Force vectors for an axis equal to
 ma and solve for unknowns

Examples

$$F_{net} = F_A + F_f \quad ma = F_A + F_f$$

Lift problem

$$F_{net} = F_A + F_g \quad ma = F_A + F_g$$

$ma = F_A + mg$
(must pay attention)
to direction (sign)

$$F_{net \ y} = F_{gy} \quad ma = mg \sin \theta$$

$a = g \sin \theta$

Elevator Problem

Normal Force (upward)
is the apparent weight

- accelerating upward $F_N > F_g$
- accelerating down $F_N < F_g$
- constant speed or rest $F_N = F_g$

graphs

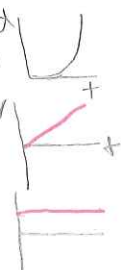
d vs t curves

v vs t slope

a vs t constant

non zero

zero



Other Force Equations

Frictional Force, Centripetal Force, Force on a Spring and

Forces on and incline ($F_{g\parallel}$ and $F_{g\perp}$)

Vocabulary:

Coefficient of Friction

$$\mu = \frac{F_f}{F_N} \text{ on Front of PRT}$$

Kinetic Friction

moving

$$\mu_s > \mu_k$$

Static Friction

at rest

Centripetal Acceleration

When direction is changing

$$a_c \text{ toward center } a_c = \frac{v^2}{r}$$

Centripetal Force

Force to center $F_c = ma_c$

$$F_c = m \frac{v^2}{r}$$

Tangential Velocity



Elongation of a spring

x (m)

Spring Constant

$$k = \frac{F_s}{x} \text{ (N/m)}$$

Spring Force

$$F_s = kx \text{ For vertical}$$

$$F_s = F_g$$

$$kx = mg$$

Parallel component of weight

$$F_{g\parallel} = F_g \sin \theta$$

$$mg \sin \theta$$

Perpendicular Component of weight

$$F_{g\perp} = F_N = F_g \cos \theta$$

Normal Force

Perpendicular to surface

$$F_N = F_g \text{ on level surface}$$

$$F_N = F_{g\perp} \text{ on incline} = mg \cos \theta$$

Angle of incline

level = 0°

vertical = 90°

Frictional Force

- Always opposes motion or net force

$$F_{\text{net}} = F_A + F_f \text{ or } F_{\text{net}} = F_A + F_f$$

$$F_f = \mu F_N$$

Only factors to influence F_f are presence of motion ($\mu_s > \mu_k$) magnitude of normal force

Static $\rightarrow$ start motion
push at rest

Kinetic $\rightarrow$ keep

Centripetal Force

Net force which causes circular path

$$F_c = ma_c \quad a_c = \frac{v^2}{r}$$

$$F_c = m \frac{v^2}{r}$$

$$v = \frac{\text{Circumference}}{T} = \frac{2\pi r}{T} \text{ + of 1 rev}$$

F_c & a_c toward center
Velocity tangent



F_c is always caused by another agent

For a car on a curve

$$F_c = F_f$$

For a planet in orbit

$$F_c = F_g$$

For an object on a string

$$F_c = F_T$$

Spring Force

$$F_s = kx \quad k \text{ is spring constant } x \text{ is stretch}$$

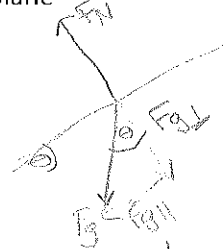
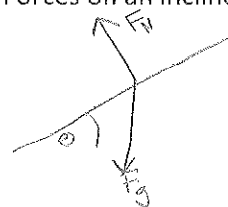
in vertical

$$F_s \text{ is weight so } F_g = mg \rightarrow F_s = F_g$$

$$kx = mg$$

One of the only forces that is not constant as F_s increases x increases

Forces on an inclined plane



$$F_g = \sqrt{F_{g\perp}^2 + F_{g\parallel}^2}$$

$$\theta = \tan^{-1} \left(\frac{F_{g\parallel}}{F_{g\perp}} \right)$$

$$F_{g\perp} = F_g \cos \theta = mg \cos \theta$$

$$F_{g\parallel} = F_g \sin \theta = mg \sin \theta$$

$\perp$ in equilibrium $F_N = F_{g\perp}$
|| axis depends on other forces

For frictionless

$$F_{\text{net}} = F_{g\parallel} = mg \sin \theta$$

$$ma = mg \sin \theta \quad a = g \sin \theta$$

For equilibrium with friction

$$F_{\text{net}} = F_{g\parallel} + F_f$$

$$-F_f = F_{g\parallel} \quad F_f = mg \sin \theta$$

(Pay attention to starting in kg or N)

Momentum and Impulse

Conservation of momentum

Momentum p (kg m/s)

$$p = mv$$

Conserved Quantity

Inertia

- Mass

- Resistance to change in condition

Impulse J $\Delta p = J$
 $J = Ft$

Force (Average Force)

Newton's 3rd Law

Conservation of Momentum
 $p_{\text{before}} = p_{\text{after}}$

Elastic Collision
 Stay separate

Inelastic collision
 Join

Separation/Explosion
 Separate from rest

Rate of change of momentum
 $\frac{\Delta p}{t} = F$

Change in momentum

Recoil

Helpful Hints Momentum and Impulse are Vectors
 On the day of the test when given the OK add the following to PRT

$$J = F_{\text{net}} t = \Delta p = [m \Delta v] \quad \text{and} \quad Ft = m \Delta v \quad F = \frac{\Delta p}{t}$$

Relationship between Momentum and Impulse

Any change in momentum is caused by an impulse
 $F_{\text{net}} = ma$ mean $F_{\text{net}} = \frac{m \Delta v}{t} = \frac{\Delta p}{t}$ (Force acting for a time)

Force and time have an inverse relationship for a constant Δp .

If two objects with the same v_i are brought to rest, they experience the same Δp & J . The net force will be different
 F or Ft (concept behind airbag)

Conservation of momentum in collision $p_{\text{before}} = p_{\text{after}}$ (ON PRT)

Elastic objects remain separate
 $p_1 + p_2 = p_1' + p_2'$
 $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$
 (If velocity is zero p is zero & can be eliminated from equation)
 momentum is a vector so direction must be taken into account

Inelastic objects initially separate, join together

$$p_1 + p_2 = p_{1+2}$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

usually larger mass so final v is less

$$1 + 2 = 12$$

Separation/Explosion objects initially together separate
 (at rest) usually

$$0 = p_1 + p_2$$

$$0 = m_1 v_1 + m_2 v_2$$

$$m_1 v_1 = -m_2 v_2$$

Recoil

equations are given for each object

Work, Energy, Power Conservation of Energy

VOCABULARY

Work = Energy (Joules)
 $W = Fd = \Delta E$

Energy (J)

KE, PE, PEs or Q

Power = Rate of energy or work

$$P = \frac{W}{t}$$

Work against Gravity

$$W = F_g d = mgh = PE$$

Work on a frictionless surface

$$W = F_{net} x d = mad = KE$$

Work on a spring

$$W = \frac{1}{2} F_s x = \frac{1}{2} kx^2 = PEs$$

Work against friction

$$W = F_d d = Q$$

Potential energy

Energy associated with vertical position

Kinetic energy

Energy associated with motion

Elastic (Spring) Potential Energy

Energy stored in a spring
 PEs

Internal Energy

Energy related to friction

Spring constant

$$k = \frac{F_s}{x} = \frac{N}{m}$$

Total (Mechanical) Energy

Change in Energy

Conservation of Energy

Relationship between Work, Energy and Power

Work done is equal to energy gained

$$W = Fd = \Delta E \quad \text{measured in Joules or Nm or kg m}^2/\text{s}^2$$

F_{vector} must agree with direction of d if force applied at angle

Power is the rate of Work or Energy

$$P = \frac{W}{t} = \frac{\Delta E}{t} = \frac{Fd}{t} = Fv \quad \text{measured in Watts}$$

Work Energy Theorem

Work on a horizontal frictionless surface = KE

$$W = F_{net} d = mad = \frac{1}{2} mv^2 \quad (\frac{1}{2} = 2ad \text{ so } ad = \frac{1}{2} v^2)$$

Work against gravity = ΔPE Constant force so Fd

$$W = F_g d = mgh$$

Work on a spring = ΔPEs (remember Force $\uparrow$ with stretch so you need avg force with springs)

$$W = \frac{F_s}{2} x = \frac{1}{2} kx^2$$

Work against friction = Q

$$W = F_d d = Q \quad \text{also equals difference between ideal \& real}$$

Conservation of Energy (remember bar graphs)

Unless other work is done Total mechanical energy is constant throughout problem (KE + PE)

$$E_{T \text{ top}} = E_{T \text{ Bottom}} \quad E_T = KE + PE \quad (\text{Frictionless})$$

$$E_T = KE + PE + Q \quad (\text{Friction/Heat})$$

	PE + KE	E_T
Top		
Mid		
Bottom		

Any loss in PE becomes KE

without friction E_T remains constant

KEY to problem solving is finding point where all energy is a single form ex: $PE = E_T$ at top

$$KE = E_T \text{ at bottom}$$

Negative Work, Energy lost to friction, Internal energy

When friction or resistance is present

Negative Work takes away from total mechanical energy

-Internal energy becomes heat

$$\text{use } E_T = PE + KE + Q$$

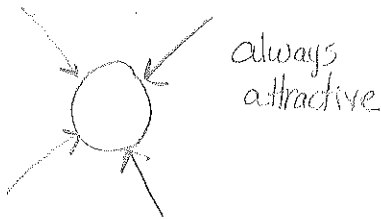
$$Q = E_T - (PE + KE)$$

Gravitational, Electric and Magnetic Fields

Gravitational Fields

Uniform Field (Surface of Planet)

$$F_g = mg \quad g = \frac{F_g}{m} \left(\frac{N}{kg} \right)$$



always attractive

$$F_g = mg$$

Force between Masses

$$F_g = \frac{G m_1 m_2}{r^2} \quad G = 6.67 \times 10^{-11} \frac{N m^2}{kg^2}$$

Whatever happens to r ; opp^2 to F_g

Whatever happens to F_g ; opp to r

$$Ex \text{ if } r \times 3 \quad F_g \div 9$$

$$\text{if } F_g \div 9 \quad r \times \sqrt{9} \times 3$$

$$\text{if } r \times 2 \quad F_g \div 4$$

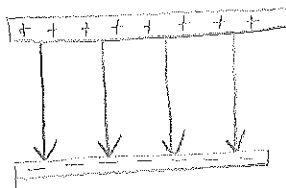
$$\text{if } F_g \times 2 \quad r \div \sqrt{2}$$

$$m_{electron} = 9.11 \times 10^{-31} kg$$

$$m_{proton} = 1.67 \times 10^{-27} kg$$

Electric Fields

Uniform Field (Parallel Plates)



$$F_e = E q$$

$$E = \frac{F_e}{q} \left(\frac{N}{C} \right)$$

Force between Charges

$$F_e = \frac{k q_1 q_2}{r^2} \quad k = 8.99 \times 10^9 \frac{N m^2}{C^2}$$

$$q_{electron} = 1.6 \times 10^{-19} C$$

F_e = direct to q

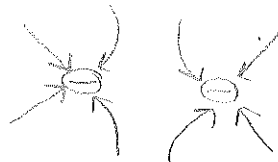
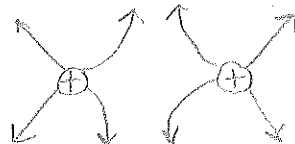
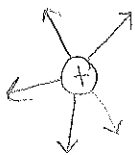
F_e = inverse square to r

r = inverse square root to F_e

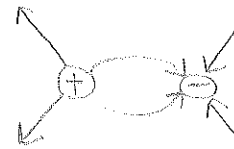
$r \times 2$ means $F_e \div 4$

$F_e \times 2$ means $r \div \sqrt{2}$

Fields around charges

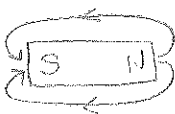


Work done against field increases potential energy

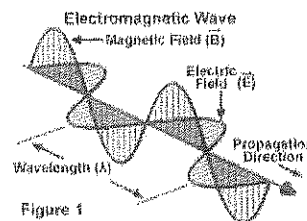


lines with arrows away from $\oplus$ toward $\ominus$

Fields around magnets



The origin of a magnetic field is the oscillation (acceleration) of a charge



Electric charge $\rightarrow$ electric field

moving electric charge $\rightarrow$ electric & magnetic field

accelerating charge $\rightarrow$ photon

$\rightarrow$ EM wave

Charge and Movement of Electrons

Proton

$$q = +e = 1.6 \times 10^{-19} \text{ C}$$

Neutron

$$q = 0 \text{ e}$$

Electron

$$q = -e = -1.6 \times 10^{-19} \text{ C}$$

Sub-atomic

part of the atom

Elementary Particle

Smallest possible
stable particle

Elementary Charge

Smallest possible
stable charge

Electromagnetic force

Force between charged
particles

op charges attract
like charges repel

Strong Force

Force between quarks

Conductor

materials which allow
electrons to flow
easily

Insulator

materials which prevent
electrons from flowing
easily

Conduction

movement of electrons
by contact

Induction

temporary imbalance of
electrons due to
repulsion

Electroscope

Device used to
observe charge

Converting between elementary charge and coulombs

$$1e = 1.6 \times 10^{-19} \text{ C}$$

multiply into $\times 1.6 \times 10^{-19}$
divide out $\div 1.6 \times 10^{-19} \text{ C}$

$$\text{Ex: } 22e \times \frac{1.6 \times 10^{-19} \text{ C}}{1e} = 3.52 \times 10^{-18} \text{ C}$$

$$8 \times 10^{-17} \text{ C} \times \frac{1e}{1.6 \times 10^{-19} \text{ C}} = \frac{8 \times 10^{-17} e}{1.6 \times 10^{-19}} = 500e$$

Possible charge on an object

elementary charge must be a whole # of e

- Cannot have a partial elementary charge

- Can have a partial coulomb as long as it doesn't
split on "e"

→ Divide charge in coulombs by $1.6 \times 10^{-19} \text{ C}$ and make
sure it is a whole #.

Loss or gain of electrons

ONLY Electrons are transferred

If electrons are lost, the charge on an
object becomes less negative or more +

If electrons are gained, the charge on an
object becomes more negative or less +

- Charge is conserved - The net charge remains
constant
If one object loses the other object
gains

Conductor vs. Insulator

Charge moves easily in a conductor
by contact.

Charge spreads out over the conductor
the charge on each object
is found by $\frac{\text{total charge}}{\# \text{ of objects}}$

Charge moves in an insulator but only
over a short distance
(polarized)

Charged objects attract opposite AND Neutral

Electric Field, Electrical Energy (work), Potential Difference, Current and Resistance

Electrical energy

$$W = Vq = (\text{Joules}) = Vq$$

$$1eV = 1.6 \times 10^{-19} \text{ J}$$

Electric field strength

$$E = \frac{F_e}{q} = (N/C)$$

Electric Field Strength

$$E = \frac{F_e}{q} = (N/C)$$

Electric Field is uniform between plates

Electric Field weakens with the square of distance from a point source

Work = Electrical Energy

$$W = Vq = (V)(C) = J$$

Potential Difference

$$V = \frac{W}{q} = (J/C) \text{ or } (V)$$

Work on a charge (work by the field or against the field)

If like charges are pushed together (against field) or if opposite charges are pulled apart work is done.

If like charges are allowed to repel (move with field) or opposite charges are pulled together the field is doing the work.

Potential Energy (qEd) becomes kinetic energy

Current

$$I = \frac{q}{t} = (C/s = \text{Ampere})$$

Current

If charge is given in electrons convert to Coulombs (multiply by 1.6×10^{-19})

$$\text{Ampere} = C/s$$

Power Rate of Energy

$$P = \frac{W}{t} = VI = I^2 R = \frac{V^2}{R}$$

Measured in Watts = J/s

For graphs of power, you must identify dependent, independent variable and the 3rd thing held constant.

Resistance Ω

$$R = \frac{V}{I}$$

$$R = \frac{V}{I}$$

Ohm's Law

$$R = \frac{V}{I} \quad V = IR$$

$$I = \frac{V}{R}$$

Power Law

$$P = \frac{W}{t} = VI = I^2 R = \frac{V^2}{R} = (W = J/s)$$

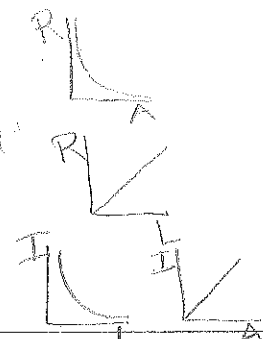
Resistance in a wire

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} \quad I = \frac{V}{R} \quad \text{so} \quad I = \frac{VA}{\rho L}$$

Low Resistance $\rightarrow$ "Short, Fat, Cold, Gold"

If temp $\uparrow$ Resistance $\uparrow$ not in an equation

$$\rho \text{ of nichrome} = 150 \times 10^{-8} \Omega \cdot m$$



Ohm's Law

$$V = IR$$

If a question says "obeys Ohm's Law" it means Resistance is constant.

Ohm's Law applies to each component and the entire circuit

Electric Circuits

Ammeters Symbol 
measures Current

Set-up in-line (series) with a component to be measured. Set up after first branch or total current

Voltmeters 

Voltmeters measure potential difference; set up in parallel.

Wires

- Conducting path for current
- Resistance in wires is zero in a circuit.

Series Circuits

Single Path for current to flow

Current equal throughout

$$I_T = I_1 = I_2 = I_3$$

Voltage split

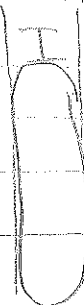
$$V_T = V_1 + V_2 + V_3$$

Resistance Sums

$$R_{eq} = R_1 + R_2 + R_3$$

VIRP Charts (Series)

Ohm's Law applies to each row

	V	I	R	P
R_1				
R_2				
R_3				
R_{eq}				

Parallel Circuits

Alternate paths for current to flow

Current split between paths

$$I_T = I_1 + I_2 + I_3$$

Potential Difference equal in each branch

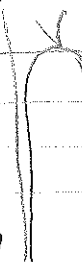
$$V_T = V_1 = V_2 = V_3$$

R_{eq} is Inverse of sum of inverses

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The addition of another resistor reduces R_{eq} & increase total current

VIRP (Parallel)

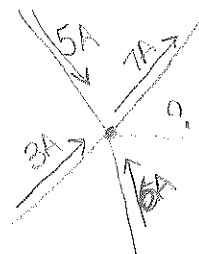
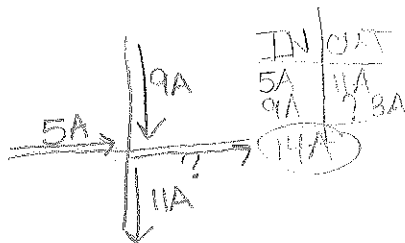
	V	I	R	P
R_1				
R_2				
R_3				
R_{eq}				

The addition of a resistor does not impact existing resistors
- Lowers R_{eq} but increases I

Junction Rule and other miscellaneous circuit

The current that enters a junction of wires must exit the junction

Ex:



IN	OUT
5A	7A
3A	?
6A	7A
	(14A)

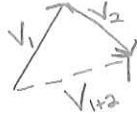
Constructive interference
 0° or 360° Δ

destructive Interference
 or node
 $180^\circ - \frac{1}{2}\Delta$

Questions involving Angles

Adding Vectors

Head to Tail



Tail to Tail



Parallel Vectors

0° difference



180° difference



Projectiles at Angles

Max Range (d_x)

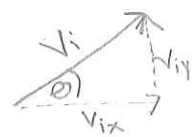
Greatest d_x is 45°
 or angle closest to 45°

Angles equal distance from 45°
 will result in same d_x

ie: 40° & 50° (45°)

30° & 60° (45°)

35° & 55° (45°)



$$v_{ix} = v_i \cos \theta \text{ (constant v)}$$

$$v_{iy} = v_i \sin \theta \text{ (a = g = 9.8 m/s}^2\text{)}$$

$$t = \frac{2v_{iy}}{g}$$

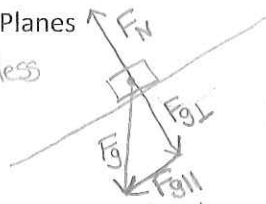
Max Time of flight

Greatest time of flight
 is the highest angle
 greatest v_{iy} highest θ

$$\theta = \sin^{-1}\left(\frac{A_y}{A}\right) \quad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Inclined Planes

Frictionless



$$F_N = F_{g\perp} \quad F_{net} = 0$$

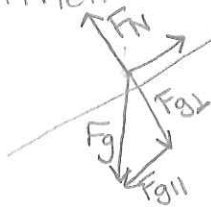
$$F_{net} = F_{g\parallel}$$

$$ma = mg \sin \theta$$

$$a = g \sin \theta$$

Normal is perpendicular to surface

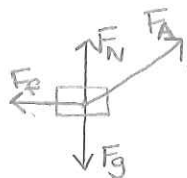
With Friction



$$F_{net} = F_{g\parallel} + F_f$$

(Be careful with sign)

Forces applied at an angle (horizontal surface) If at equilibrium $F_{net} = 0$
 If accelerating $F_{net} = ma$



$$F_{netx} = F_{Ax} + F_f$$

$$F_{nety} = F_{Ay} + F_N + F_g$$

$$F_{Ax} = F_A \cos \theta$$

$$F_{Ay} = F_A \sin \theta$$

$F_g = F_N$ on a level surface without
 other applied forces

$F_g = F_N + F_{Ay}$ on a level surface
 with applied force

Reflection and Refraction

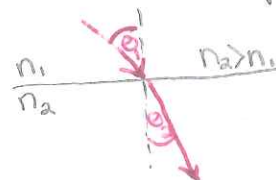
$\theta_i = \theta_r$ Law of reflection
 angles measured from Normal
 line



$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Snell's Law

refraction



ray crosses
 from
 Q_2 to Q_4
 can't bounce
 off normal

Converting between Units

Converting between elementary charge and charge in coulombs

elementary charge $\rightarrow$ Coulomb's
multiply by $\frac{1.6 \times 10^{-19} \text{C}}{1e}$

$$1e = 1.6 \times 10^{-19} \text{C}$$

Coulombs $\rightarrow$ elementary charge
divide by $1.6 \times 10^{-19} \text{C}$
 $6.4 \text{C} \times \frac{1e}{1.6 \times 10^{-19} \text{C}} = 4 \times 10^{19} e$

You can't have a fraction of an e

Converting between energy in joules and energy in electronvolts

$$1 \text{eV} = 1.6 \times 10^{-19} \text{J}$$

electron volt is the energy required to move 1 elementary charge through a potential difference of 1V

$$W = qV$$

$$= CV = \text{Joule}$$

or eV

You can have a fraction of an eV (since you can have a fraction of a V)

$$\text{into J} \times \frac{1.6 \times 10^{-19} \text{J}}{1 \text{eV}}$$

$$\text{out of J} \times \frac{1 \text{eV}}{1.6 \times 10^{-19} \text{J}}$$

Converting between mass and energy

Universal (atomic) mass units to energy

If mass is in $u \times 9.31 \times 10^2 \text{MeV}$
($E = mc^2$ already done)

$$\text{Ex: } 0.036u \times \frac{9.31 \times 10^2 \text{MeV}}{1u} = 33.5 \text{MeV}$$

mass in kg to energy

mass in kg to energy in J
use

$$E = mc^2$$

$$1 \text{MeV} = 1 \times 10^6 \text{eV}$$

$$1 \text{MeV} \times \frac{1 \times 10^6 \text{eV}}{1 \text{MeV}} \times \frac{1.6 \times 10^{-19} \text{J}}{1 \text{eV}} = 1.6 \times 10^{-13} \text{J}$$

If it relates to movement of electrons in hydrogen & mercury
convert Joules to eVs

Converting between mass in kg and pounds (not on Regents but helpful for estimation)

$$1 \text{kg} = 2.2 \text{lbs}$$

$$1 \text{in} = 2.54 \text{cm}$$

$$1 \text{oz} = 28.35 \text{g}$$

$$1 \text{mi} = 1.609 \text{km}$$

$$1 \text{km} = .62 \text{mi}$$

$$\frac{65 \text{mi}}{\text{hr}} \times \frac{1609 \text{m}}{1 \text{mi}} \times \frac{1 \text{hr}}{3600 \text{s}} = 29 \text{m/s}$$

$$\boxed{65 \text{mi/hr} = 29 \text{m/s}}$$

$$10 \text{m/s} \times \frac{1 \text{mi}}{1609 \text{m}} \times \frac{3600 \text{s}}{1 \text{hr}} = 22.3 \text{mi/hr}$$

"Tricky" but simple questions

A plane flying with a horizontal velocity of 80 m/s drops a 3kg box from a height of 1000m. What is the horizontal velocity of the box just before it hits the ground?

Horizontal Velocity does not change

80 m/s

An astronaut with a mass of 80kg travels from Earth, to a point 4 Earth radii away from the center of the Earth, what is the mass of the astronaut at this location?

Mass does not change with gravitational field

Lightbulb A and B are connected in series to a 20V source of potential difference. If the current through lightbulb A is 2amperes, what is the current through lightbulb B?

Current remains constant in series

What is the max and min resultant?

0° between vectors means same direction
→ Max Resultant

180° between vectors mean opposite direction
→ Min Resultant

Any question involving vectors and change in direction

Pay attention to direction
→ assign $\oplus$ & $\ominus$

east positive x west negative x
north positive y south negative y

Miscellaneous

- Newton's 3rd Law - IF two objects push on each other
The forces are equal (forces act on each object in different diagrams)
Forces are equal
acceleration depends on mass
- Rate of energy equals Power (measured in Watts)
- Inertia is mass