

Topic 3A: Newton's Laws, Force Vectors and Equilibrium Equations

Skill 22: Part Two-Applying equations to Horizontal, Vertical and Inclined Planes Scenarios

- Forces are vector quantities, meaning direction is important. Specifically, you must understand how each force in a problem aligns with each axis and keep in mind the sign (+ or -) for each force. Only forces from the same axis can be combined by simple addition.

For object on a level surface (horizontal motion)

- Axes are defined as "x" & "y"
- For forces applied at an angle use equations

$$F_{Ax} = F_A \cos \theta \quad \theta = \tan^{-1} \left(\frac{F_{Ay}}{F_{Ax}} \right)$$

When force is horizontal (or parallel to level ground) $\theta = 0$
So $F_{Ax} = F_A$ as θ increases F_{Ax} decreases

- For any axis of motion you must determine
 - If forces are balanced; ie equilibrium $F_{net} = 0, a = 0$
ie constant speed or rest
 - If forces are unbalanced; ie accelerating, $F_{net} = ma, F_{net} = m \frac{\Delta v}{t}$
ie speeding up; slowing down changing direction

- To solve for unknowns you combine the above into a single equation where "The sum of forces on any axis" is set equal to "zero or ma "

Examples

Objects moving or at rest on a level surface

$$\sum F_x = F_{Ax} + F_f \quad \text{At equilibrium } F_{Ax} + F_f = 0$$

If force is applied horizontal then $F_{Ax} = F_A$ ($F_{Ax} = F_A \cos 0$)

If multiple forces are applied then add 1st $\theta = 0$

Objects on a vertical axis

$$\sum F_y = F_A + F_g \quad \text{where } F_g = mg \quad \text{always downward}$$

If $F_A = F_g$ object is at rest or constant velocity

If $F_A > F_g$ object is accelerating upward

If $F_A < F_g$ object is accelerating downward

F_A is the upward force. It can be represented as F_T for a rope or chain or as F_N for an elevator

* The net force is the imbalance (difference) between F_A & F_g

For objects on a vertical axis

- F_g is always downward
- F_N is upward from a surface
- F_f & F_A represent force from a rope, chain, etc
- At rest or constant Speed $F_N = F_g$ (elevator)
 $F_A = F_g$ (rope)

For objects on an inclined plane

Axes are defined parallel to plane " \parallel " or perpendicular to plane " \perp "

" \perp " - F_N & $F_{g\perp}$ are equal but opposite

" \parallel " - $F_A, F_f, F_{g\parallel}$ are on axis of motion



With net force $F_{Ax} + F_f = ma$
present

$F_{Ax} = F_f$ constant speed or rest

$F_{Ax} > F_f$ speeding up due to applied force

$F_{Ax} < F_f$ slowing down due to Friction

Objects on an inclined plane

If $F_f = 0$ (frictionless incline)
 $F_{g\parallel} = F_{g\sin \theta} = mgs \sin \theta$
or $ma = mgs \sin \theta$
so $a = g \sin \theta$

With Friction or other applied force

$$\sum F_{\parallel} = F_{g\parallel} + F_A + F_f$$

$$\sum F_{\perp} = F_{g\perp} + F_N \quad \text{since } \perp \text{ is at equilibrium}$$

$$F_N = -F_{g\perp} = F_g \cos \theta = mg \cos \theta$$