Name: _____ Date: _____

Introduction to Geometry

INTRO REVIEW #1

Factoring – there are 3 methods of factoring:

1. GCF – (Greatest Common Factor): the largest amount that can be divided out of all terms of an expression.

   Ex: Factor out the GCF: $16x^3y^2 + 12xy^5$ GCF: $4xy^2$ Factored: $4xy^2(4x^2 + 3y^3)$

2. Difference of Perfect Squares: Everything must be perfect squares, and must be separated by a subtracting sign.

   Ex: $25x^2 – 49y^2 = (5x + 7y)(5x – 7y)$

3. Multiply/Add: In a trinomial of the form $ax^2 + bx + c$ ($a \neq 0$), factors have a sum = b, and a product = c

   Ex: Factor: $x^2 – 3x – 10$ since $– 5 \bullet 2 = - 10$ and $-5 + 2 = - 3$, the factors are $(x – 5)(x + 2)$

Example # 1: Factor the following:

a.) $28a^2 - 14a$ b.) $49x^2 - 81y^2$ c.) $x^2 - 5x - 14$

Factoring Completely: Factor out the GCF first, then use method # 2 or # 3 to factor what is left.

***Example # 2*:** Factor each of the following completely:

a.) $3x^2 - 3x - 6$ b.) $b^3 - 9b$ c.) $2a^2 - 2a - 12$

d.) $x^3 + 5x$ e.) $2xy^2 + 8x^2y$ f.) $x^2 + x$

Simplifying rational expressions - Factor First!

Example # 4: Which of the following is equivalent to $\frac{6x^2-2xy}{9x^2-y^2}$ in simplest form?

(1) $\frac{-2x}{3x+y}$ (2) $\frac{-2x}{3x-y}$ (3) $\frac{2x}{3x-y}$ (4) $\frac{2x}{3x+y}$

Example # 5: Express each of the following in simplest form.

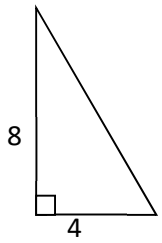
(a) $\frac{2x-10}{x^2-2x-15}$ (b) $\frac{x^3-9x}{x^2+3x}$

(c) $\frac{x^2-4}{2x-4}$ (d) $\frac{x^2+2x-3}{x^2+3x}$

MIXED REVIEW

1. What is the surface area of a circle, to the *nearest hundredth*, whose radius is 11 ?

2. What is the length of the hypotenuse of the given right triangle in *simplest radical form?*



INTRO REVIEW #1

HOMEWORK

1. Factor the following:

   a.) $6x^2 + 9x$

   b.) $x^2 - 5x - 6$

   c.) $18x^2y^3 + 24x^5y^5$

   d.) $144 - 25x^2$

2. Factor the following completely:

   a.) $x^3 - 4x^2 - 12x$

   b.) $5x^2 - 45x + 90$

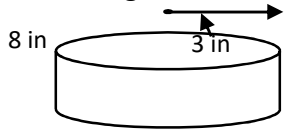
   c.) $8x^3 - 72x^2$

   d.) $10p^2 + 10p - 20$

3. What is the volume, to the *nearest tenth of a centimeter*, of a sphere whose diameter is 18 centimeters?

4. What is surface area of a circle, to the *nearest tenth*, whose radius is 6?

5. Find the volume of the cylinder represented in the diagram below. Leave your answer in terms of π.



6. Simplify the following rational expressions:

a.) $\frac{3x^2+3x-36}{x^2-5x+6}$

b.) $\frac{x^2-25}{6x-30}$

c.) $\frac{3x^2+15x}{6x^2-150}$

d.) $\frac{9x^2-1}{6x^2-2x}$

7. Towns *A* and *B* are 16 miles apart. How many points are 10 miles from town *A* and 12 miles from town *B*?

(1) 1 (2) 2 (3) 3 (4) 0

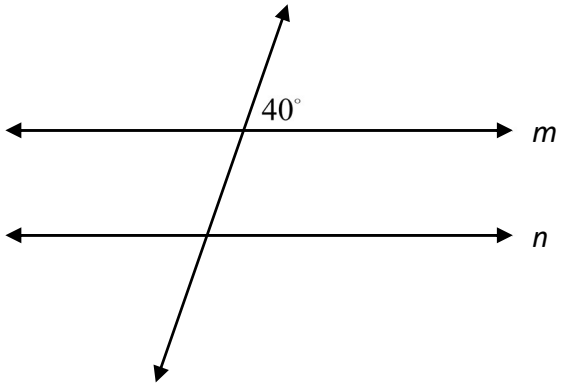
8. Through a given point, *P*, on a plane, how many lines can be drawn that are perpendicular to that plane?

(1) 1 (2) 2 (3) More than 2 (4) None

INTRO REVIEW # 2

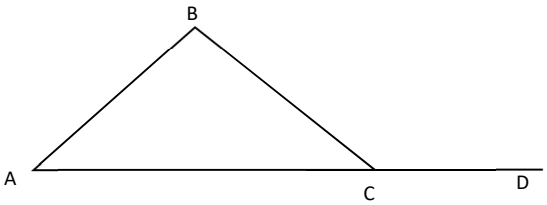
Parallel lines

Example # 1: Given $m \parallel n$, find the measure of the rest of the angles.

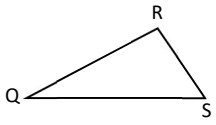


Triangles

Example # 2: In the diagram of ΔABC, $\overline{AB} \cong \overline{BC}$, and $m\angle A = 40°$. Find $m\angle B$, $m\angle BCA$, and $m\angle BCD$.

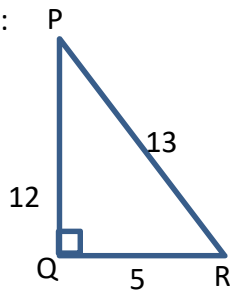


Example # 3: The ***degree*** measures of the angles of ΔQRS are represented by x, $5x$, and $3x - 45$. Find the value of x.



SOH-CAH-TOA

Example # 4:In the following diagram, find each of the following ratios:



$\sin P =$ $\cos Q =$

$\tan Q =$ $\tan P =$

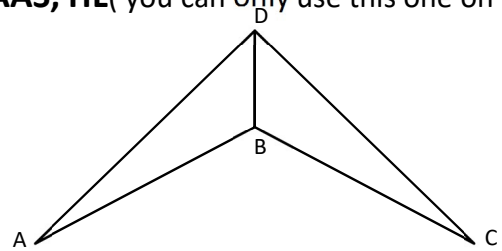
$\cos P =$ $\sin Q =$

Congruent Triangles

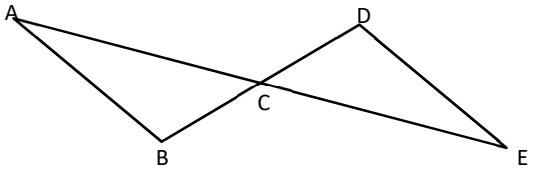
Example # 5: Given the following information, state which triangle congruence theorem can be used to prove the triangles are congruent.

CONGRUENCE THEOREMS: **SSS, SAS, ASA, AAS, HL**(you can only use this one on *right* triangles.)

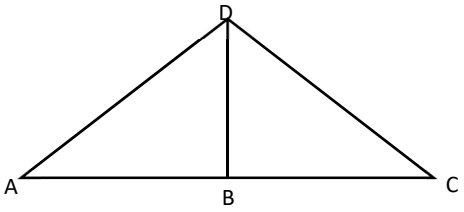
(a) Given: $\overline{AD} \cong \overline{DC}$, $\overline{BD}$ bisects $\angle ADC$



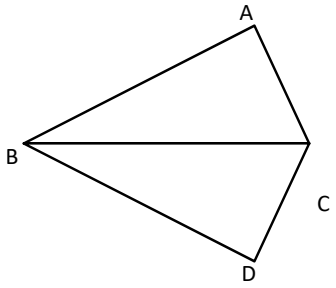
(b) Given: $\angle B \cong \angle D$, C is the midpoint of $\overline{BD}$.



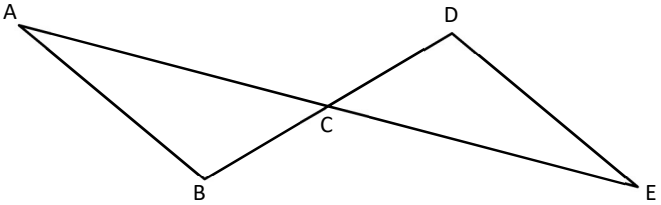
(c) Given: $\overline{AD} \cong \overline{DC}$, $\overline{BD}$ bisects $\overline{AC}$



(d) Given: $\angle A$ and $\angle D$ are right angles, $\overline{AC} \cong \overline{CD}$



(e) Given: $\angle A \cong \angle E$, C is the midpoint of $\overline{BD}$.



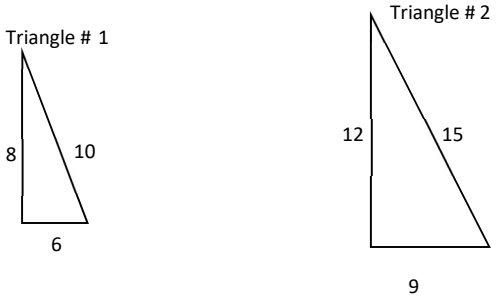
Similar Triangles: If two triangles are *similar* to one another, their sides are in **proportion** to one another.

Example # 6: At the same time that a tree casts a shadow 24 feet long, a man that is 6 feet tall casts a shadow 4 feet long. Find the height of the tree.

***Example* #7:** The measures of the sides of a triangle are 4, 7, and 10 inches long. If the longest side of a similar triangle is 25 inches, find the length of the shortest side of that triangle.

Similar Triangles and their Areas: If two triangles are ***similar***, then the ratio of their areas equals the square of the lengths of any two corresponding sides.

Example # 8: Given the two following similar triangles, answer the following questions.



a.) What is the ratio of the sides?

b.) What is the area of each triangle?

c.) What is the ratio of the areas of the triangles?

Example # 9: The ratio of two corresponding sides of two similar triangles is $\frac{4}{9}$. What is the ratio of their areas?

Lengths of the Sides of a Triangle – The sum of any two sides of a triangle must be *greater* than the length of the third side.

Example # 10 : Which set of numbers represents the lengths of the sides of a triangle?

(1) { 2, 3, 1 } (2) { 5, 9, 15 } (3) { 3, 5, 7 } (4) { 9, 5, 3 }

Example # 11 : Which set of numbers cannot represent the lengths of the sides of a triangle?

(1) { 3, 7, 9 } (2) { 4, 4, 4 } (3) { 5, 5, 9 } (4) { 3, 6, 9 }

Note: Do not confuse triangle side questions with *right* triangle questions.

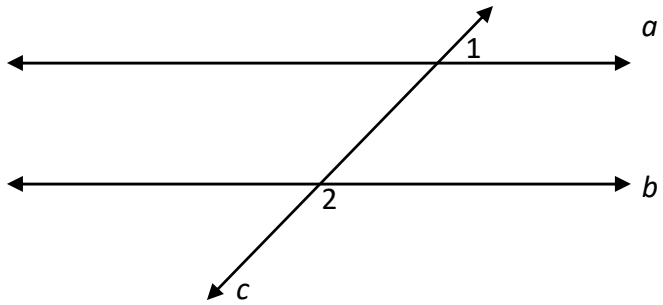
Example # 12: Which of the following represent the sides of a right triangle?

(1) { 2, 3, 4 } (2) { 7, 24, 25 } (3) { 3, 4, 6 } (4) { 5, 12, 15 }

INTRO REVIEW # 2 Homework

1. If the lengths of the legs of a right triangle are 2 and 3, then the length of its hypotenuse is:
   (1) $\sqrt{13}$ (2) $\sqrt{5}$ (3) 5 (4) 4

2. In the diagram below, lines *a* and *b* are cut by transversal *c.*



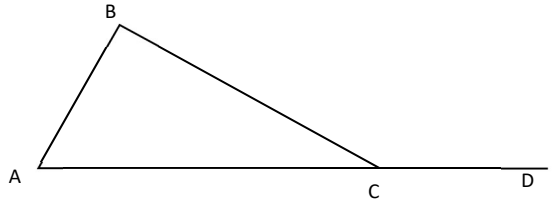
   If the measure of $\angle 1 = 41^\circ$, what is the measure of $\angle 2$?

   (1) 41° (2) 139° (3) 49° (4) 128°

3. Which set of numbers represents the lengths of the sides of a triangle?
   (1) { 4, 9, 13 } (2) { 5, 7, 11 } (3) { 3, 4, 6 } (4) { 1, 2, 3 }

4. Simplify: $\dfrac{6x^2 - 2xy}{9x^2 - y^2}$

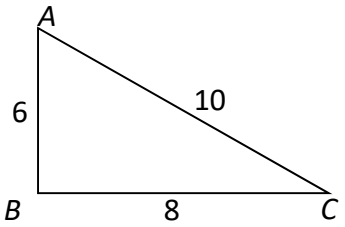
5. In the diagram below, ΔABC is shown with $\overline{AC}$ extended through point D. If $\angle BCD = 112^\circ$, $\angle BAC = 3x + 21$, and $\angle ABC = 2x + 31$, find the value for *x*.



6. In the following diagram, which of the following represents $\cos C$?

(1) $\frac{6}{10}$ (3) $\frac{6}{8}$

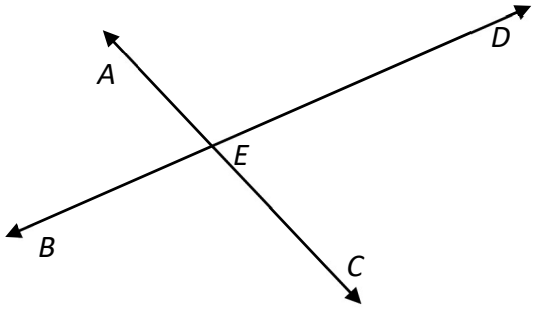
(2) $\frac{8}{10}$ (4) $\frac{8}{6}$



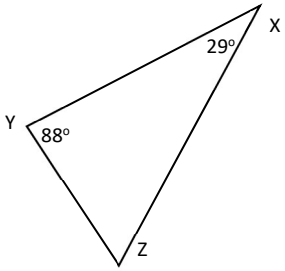
7. Given the ratios of the sides of two similar triangles, find the ratios of the areas.

a.) $\frac{5}{6}$ b.) $\frac{10}{3}$ c.) $\frac{1}{8}$ d.) 12:13

8. In the figure shown, lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ intersect at *E*, and the $m\angle AED = 116^\circ$. What is the measure of $\angle AEB, \angle BEC,$ and $\angle CED$?



9. Given ΔXYZ , which side is *shortest*? Justify your answer. (*picture is not drawn to scale*)



INTRO Review # 3

Graphing a line You must get the equation in $y = mx + b$ form.

Exercise # 1 : Put each equation in $y = mx + b$ form. Then state the line's slope and y-intercept.

a.) $5y - 2x = 10$ b.) $3y + 4x = 12$ c.) $6x - 4y = 8$

Slope - Write the slope formula _____

Exercise # 2: Find the slope of the following sets of points:

a.) (- 3, 7) and (0, - 2) b.) (6, 1) and (8, 4) c.) (- 8, 3) and (- 2, 1)

Exercise # 3: Which of the following equations represents a line parallel to the line whose equation is $3y - 2x = 12$?

(1) $y = \frac{2}{3}x + 5$ (2) $y = \frac{3}{2}x - 4$ (3) $y = -\frac{2}{3}x + 7$ (4) $y = -\frac{3}{2}x - 1$

*** Parallel lines have _____ slopes

Exercise # 4: Which of the following represents the slope of a line that is perpendicular to the line whose equation is $5y + 2x = 15$?

(1) $\frac{2}{5}$ (2) $-\frac{2}{5}$ (3) $\frac{5}{2}$ (4) $-\frac{5}{2}$

***** Perpendicular lines have _____ slopes.

Midpoint – Write the midpoint formula _____

Exercise # 5: Find the midpoint of each of the following sets of points.

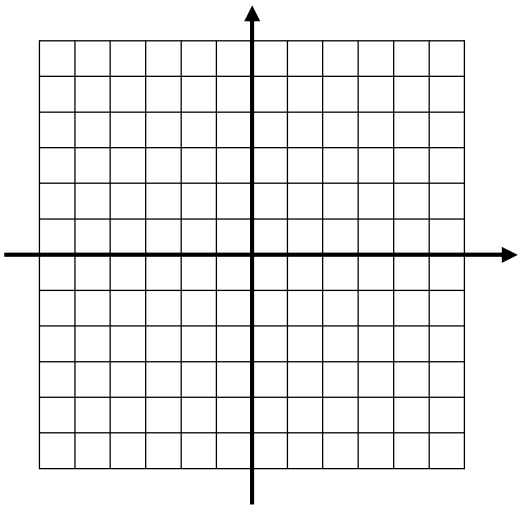
a.) (5, 8) and (- 1, 2) b.) (0, 3) and (- 6, - 1) c.) (5, - 7) and (2, - 3)

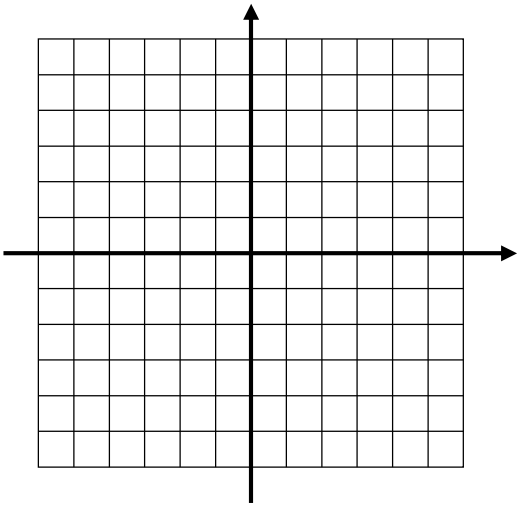
Distance - Write the distance formula _____

Exercise # 6: Find the distance between each pair of points. Express your answers in *simplest radical form*, where applicable.

a.) (1, 2) and (- 3, 0) b.) (3, 7) and (- 1, 4) c.) (0, 5) and (- 5, 0)

SCRAP GRAPH GRIDS





INTRO Review # 3 Homework

1. Find the Distance, midpoint, and slope between the following sets of points. Leave your answers in *simplest radical form*, where appropriate.

   a.) (2, 5) and (- 3, 1)

   b.) (0, 3) and (- 1, - 4)

2. Which of the following is the equation of a line parallel to the line whose equation is $2y - 3x = 6$?

   (1) $3y + 2x = 9$ (2) $3y - 2x = 4$ (3) $y = \frac{3}{2}x + 5$ (4) $y = -\frac{3}{2}x + 7$

3. Which of the following is the slope of a line perpendicular to the line whose equation is $7y + 5x = 14$?

   (1) $\frac{5}{7}$ (2) $-\frac{5}{7}$ (3) $\frac{7}{5}$ (4) $-\frac{7}{5}$

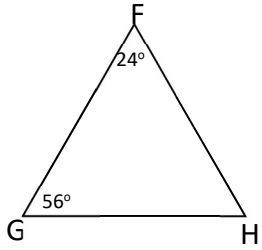
4. Simplify the rational expression $\frac{2x^2+4x-30}{6x-18}$

5. Which set of numbers represents the side of a triangle?

   (1) { 4, 8, 12 } (2) { 3, 3, 5 } (3) { 5, 5, 10 } (4) { 5, 6, 12 }

6. In a triangle, the angles measure 5x, 3x, and x – 27. Find the value of x.

7. In triangle FGH, which side is the longest? Justify your answer. (Picture is not drawn to scale)



INTRO REVIEW # 4

Reflections (flip)in the:

y – axis (x, y) → (-x, y) { negate the x } **Translation** - add

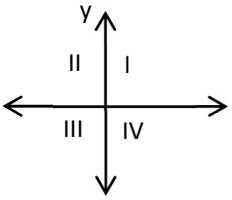
x – axis (x, y) → (x, -y) { negate the y } **Dilation** - multiply

y = x (x, y) → (y, x) { switch coordinates }

y = - x (x, y) → (-y, -x) { switch and negate }

Rotation: positive rotation - counterclockwise number) negative rotation - clockwise

Quadrants



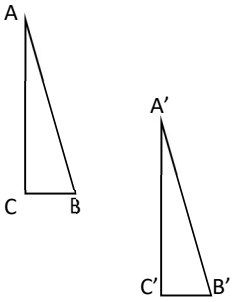
Horizontal lines: y = n (n is any number)

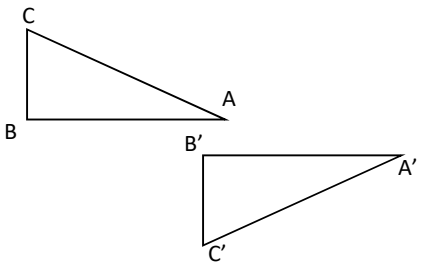
Vertical lines : x = n (n is any

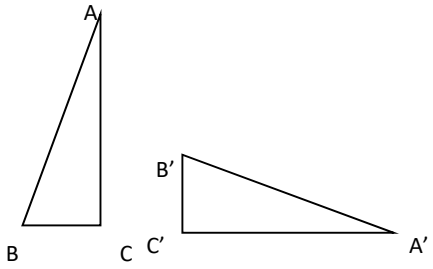
Exercise # 1: Categorize each transformation as either a reflection, translation, rotation, dilation or glide reflection.

b.)

c.)



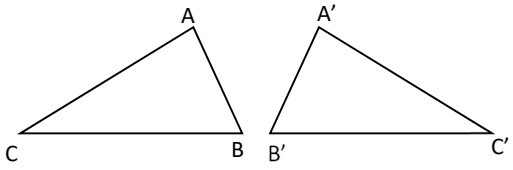


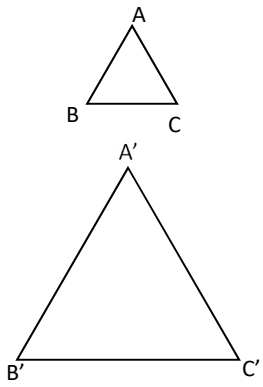


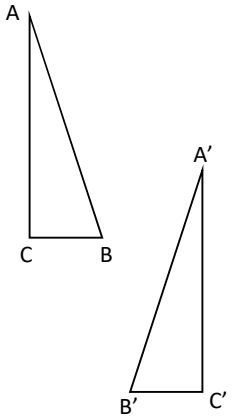
d.)

e.)

f.)







Exercise # 2: Write the image of point (- 2, 3) under the following transformations:

a.) r_{x-axis} _____ b.) $r_{y=x}$ _____ c.) $T_{-4,5}$ _____

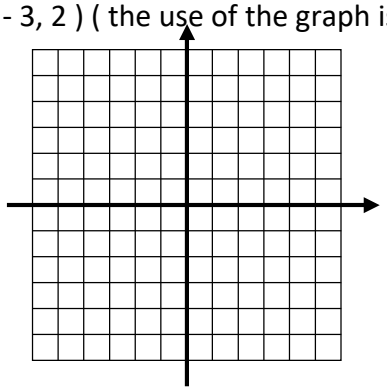
d.) D_3 _____ e.) r_{y-axis} _____ f.) $r_{y=-x}$ _____

Exercise # 3: Find each rotation for the point (- 3, 2) (the use of the graph is optional)

a.) $R_{90°}$ _____

b.) $R_{-180°}$ _____

c.) $R_{-90°}$ _____



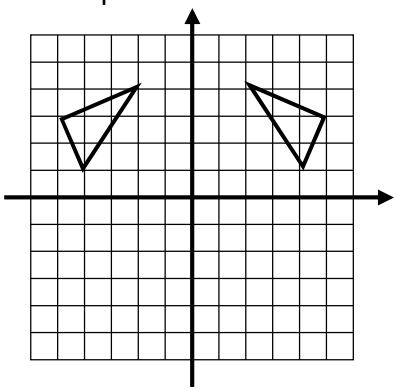
<u>Glide Reflection</u> – When applying a glide reflection, you must do the transformation closest to the point first. (Work backwards!)

Exercise # 4: Give the coordinates of the image of (-2, 3) under each of the following glide reflections:

(a) $r_{y-axis} \circ T_{0,-5}$ (b) $T_{-6,0} \circ r_{x-axis}$ (c) $T_{3,3} \circ r_{y=x}$ (d) $r_{y=-x} \circ T_{4,-4}$

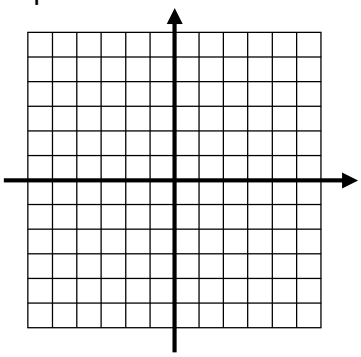
INTRO REVIEW # 4 HOMEWORK

1. Which expression best describes the transformation shown in the following diagram?

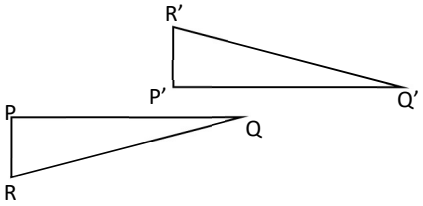


(1) Line Reflection (3) Rotation
(2) Point Reflection (4) Translation

2. What is the image of point A(-3, 2) after the composition of transformations defined by $T_{-3,-3} \circ r_{y=x}$?
(the use of the grid is optional)



3. Which transformation is used to map ΔPQR to $\Delta P'Q'R'$?
(1) Dilation (2) Rotation (3) Reflection (4) Glide Reflection



4. Simplify: $\dfrac{3x^2 - 15x - 42}{x^2 - 49}$

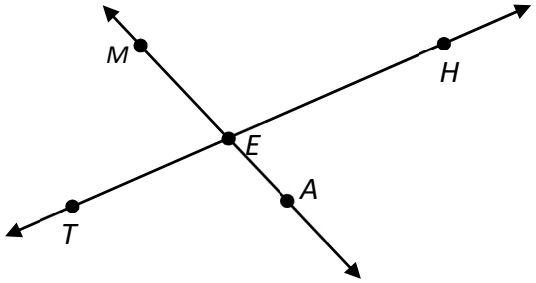
5. Given points A(-2, 6) and B(-1, -3), find the distance, midpoint and slope of segment $\overline{AB}$. Leave your answers in *simplest radical form* where appropriate.

   Distance:

   Midpoint:

   Slope:

6. In the figure shown, lines $\overleftrightarrow{MA}$ and $\overleftrightarrow{TH}$ intersect at *E*, and the $m\angle AEH = 68^\circ$. What is the measure of $\angle AET$?



7. Find each of the following transformations for the point (-7, 11).

   a.) r_{x-axis} _____ b.) $r_{y=x}$ _____ c.) $T_{5,-1}$ _____

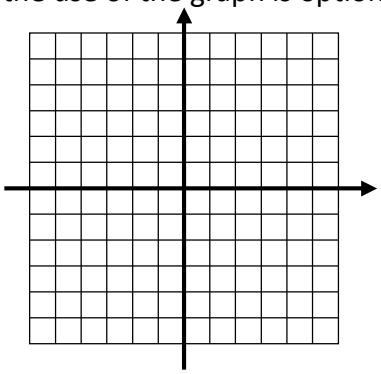
   d.) D_2 _____ e.) r_{y-axis} _____ f.) $r_{y=-x}$ _____

8. Find each rotation for the point (4, - 3) (the use of the graph is optional)

   a.) R_{90° _____

   b.) R_{-180° _____

   c.) R_{-90° _____

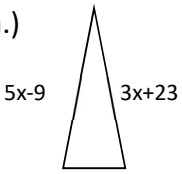


INTRO Review # 5

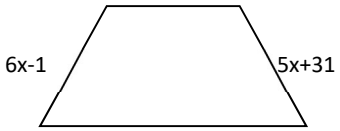
Isosceles – A figure is considered isosceles if at least two sides are equal in length.

Example # 1: The following figures are isosceles. Find the value of x.

a.)



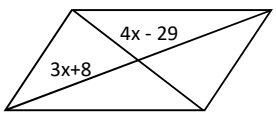
b.)

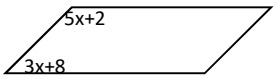


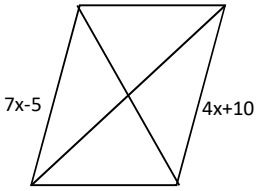
Quadrilaterals –

1.) Parallelogram – opposite sides are congruent and parallel
   - Opposite angles are congruent
   - Consecutive angles are supplementary
   - Diagonals bisect each other
2.) Rhombus – all the properties of parallelogram AND
   - All four sides are congruent
   - Diagonals are perpendicular
3.) Rectangle – all of the properties of parallelograms AND
   - All angles are right angles
   - Diagonals are congruent
4.) Square – all of the properties of all quadrilaterals above.

Example # 2: Given that the following figures are parallelograms, find the value of x.



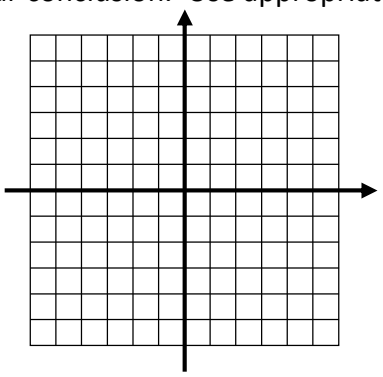




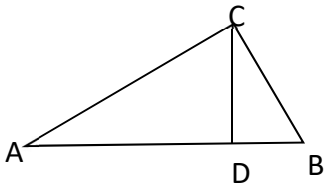
Coordinate Geometry Quadrilaterals

Example # 3: The coordinates of the vertices of Quadrilateral MATH are M(4, 6), A(2, 3), T(-4, 2), and H(-2, 5)

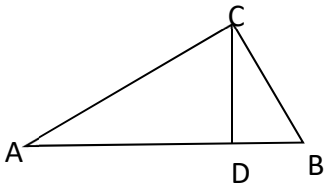
a.) Is MATH a parallelogram? SHOW how you came to your conclusion. Use appropriate formulas.



Triangles with Altitudes – if an altitude cuts the hypotenuse of a right triangle, the following proportions can be determined.



$$\frac{alt}{seg.1} = \frac{seg.2}{alt}$$



$$\frac{leg}{closer\ seg} = \frac{whole\ hyp}{leg}$$

Example # 4:

a.) If AD = 8 and DB = 2, find CD.

b.) If DB = 4 and AB = 9, find CB.

Interior and Exterior angles of polygons:

The sum of the *interior* angles of any polygon can be found by using the formula: $(n-2)180$, where n is the number of sides the polygon has.

Example # 1: What is the sum of the measures of the interior angles of a octagon?

Example # 2: What is the measure of *one* angle of a hexagon?

The sum of the *exterior* angles of any polygon is always equal to 360.

Example # 3: What is the sum of the measures of the exterior angles of an 11 sided figure?

Example # 4: What is the measure of one angle of a regular hexagon?

INTRO Review # 5 Homework

1. Given points A(3, - 5) and B(2, 7), find the distance, midpoint and slope of segment $\overline{AB}$. Leave your answers in *simplest radical form* where appropriate.

Distance:

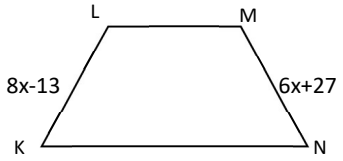
Midpoint:

Slope:

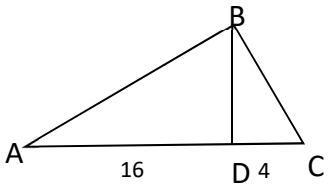
2. Which of the following is the slope of a line perpendicular to the line whose equation is 5x+4y=12?

(1) $\frac{4}{5}$ (2) $\frac{5}{4}$ (3) $-\frac{4}{5}$ (4) $-\frac{5}{4}$

3. Isosceles trapezoid LMNK is shown below with algebraic expressions for the lengths of its sides. Find the value of x.



4. In the diagram below of right triangle ABC, altitude $\overline{BD}$ is drawn to hypotenuse $\overline{AC}$



If $\overline{AD} = 16$ and $\overline{CD} = 4$, what is the length of $\overline{BD}$?

5. The coordinates of the vertices of Quadrilateral SPAM are S(1, 2), P(3, 3,), A(1, - 4), and

   M(-1, -5). Is SPAM a parallelogram? SHOW how you came to your conclusion. Use appropriate formulas.