Name: ________________________________ **UNIT 11-13 BOOK**

Quadratic Functions: Graphing a Quadratic Function
Intro to Geometry

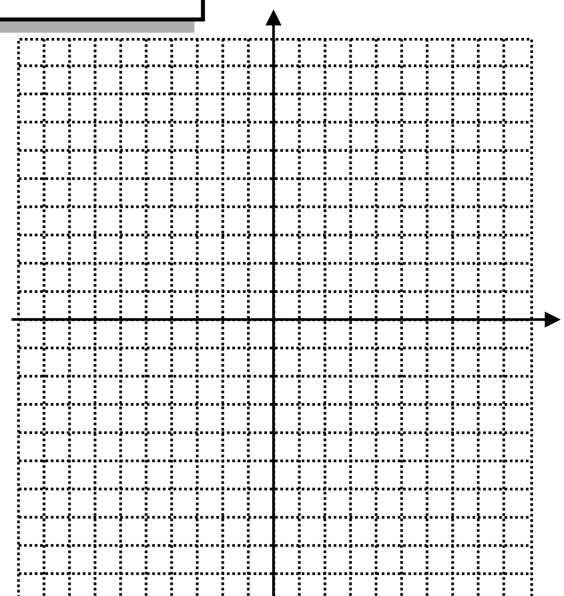
<u>AIM</u>: → ***To graph a parabola***

→ ***To identify the axis of symmetry and turning point of a parabola***

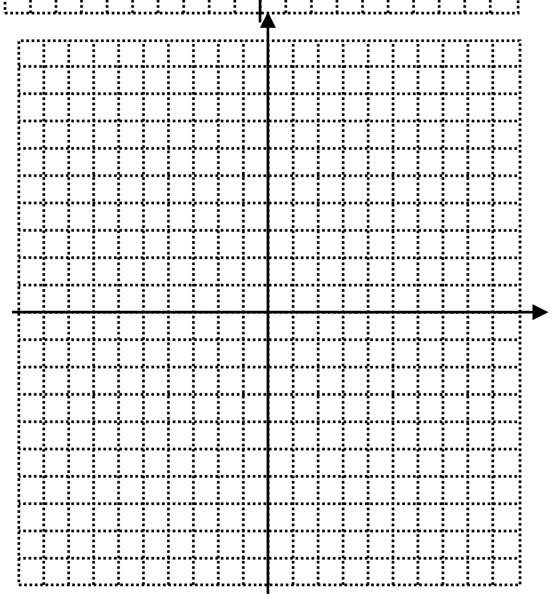
Recall: Functions can be identified by their equations.

A function of the form $y = mx + b$ creates the graph of a **<u>line</u>**.

Draw a sketch:



<u>Ex. 1</u> Graph the function $y = x^2 - 4x + 1$ for all values of *x* in the interval $-1 \le x \le 5$.



<u>Ex. 2</u> Graph the function $y = -x^2 - 2x + 4$ for all values of *x* in the interval $-4 \le x \le 2$.

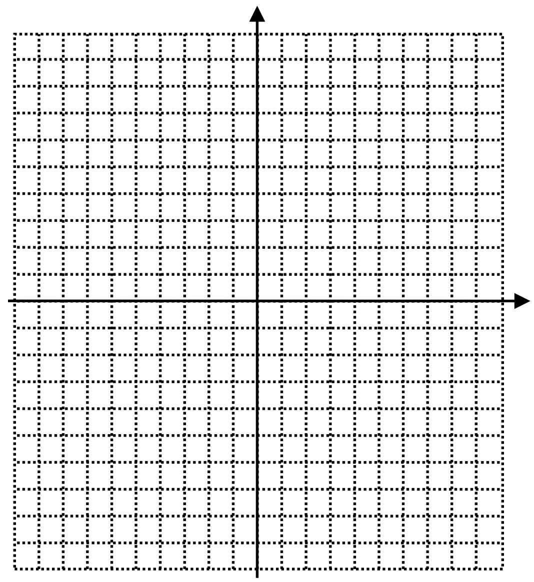
Be careful with the negative on the x squared term!!!

Features of the Quadratic Function

The vertical line that divides the parabola into two mirror images is called the **axis of symmetry**.

The maximum or minimum point on a parabola is called its **turning point**.

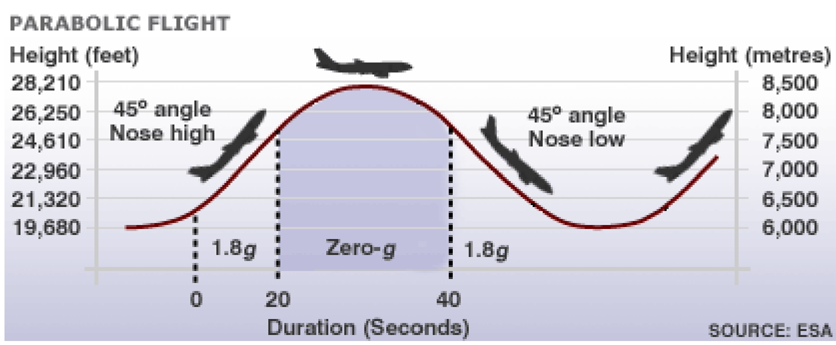
Ex. 3 Graph the function $g(x) = -2x^2 + 12x - 9$ for all values of x in the interval $0 \le x \le 6$. Graph the axis of symmetry and find its equation. Also, find the coordinates of the turning point.



Axis of Symmetry:

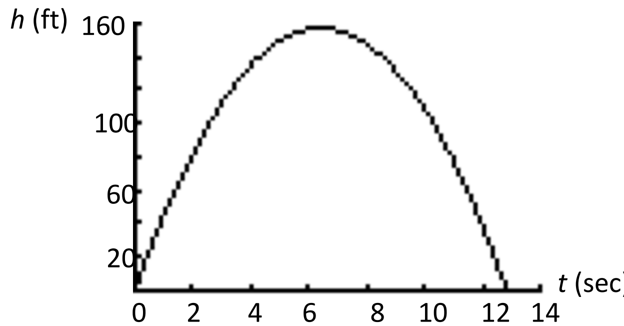
Turning Point:

Quadratic Functions have various applications in our day to day life. They are useful in modeling the flight of a golf ball, the construction of arches, and in determining the path of hanging cables that support weight. They are even used to model the flight pattern of an airplane for training future astronauts in experiences with zero gravity.



Ex. 4 The 4th of July fireworks show in Cow Town USA can be modeled by the equation $h = -3.8t^2 + 48.7t + 1.1$. The rocket will explode at its highest point.

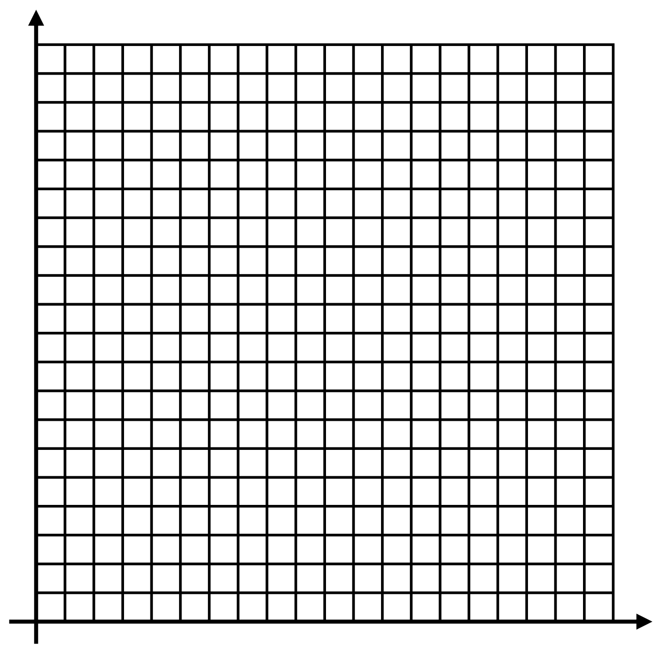
- ✦At approximately what time, *t*, will the rocket explode?
- ✦At approximately what height will the rocket explode?



Ex. 5 A ball is thrown straight up into the air from ground level. The relationship between the height (*h)* of the ball at any time (*t)* is illustrated by a graph of a parabola.

Graph the following table of values. Use increments of 10 feet for the height of the ball. The horizontal axis represents time, *t*, and the vertical axis represents height, *h*.

# Sec.	***Height***
0	**0**
1	**30**
2	**50**
3	**60**
4	**50**
5	**30**
6	**0**



a) Is this graph the graph of a function?

b) State the domain and the range.

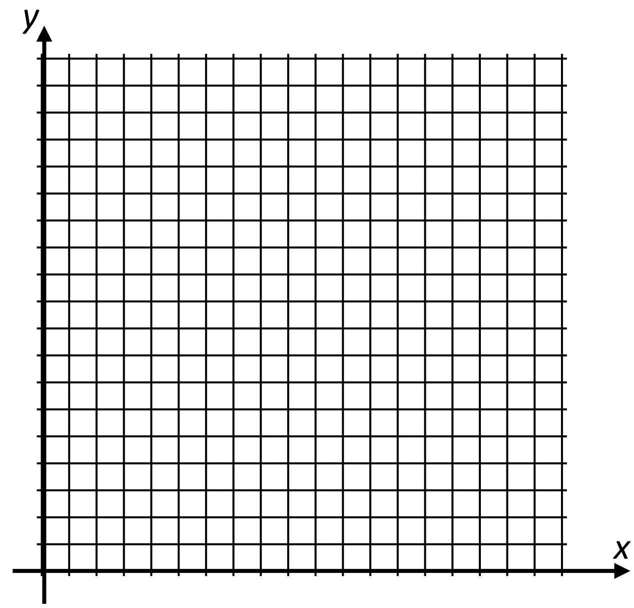
c) At what time does the ball reach its maximum height?

d) What is the maximum height of the ball?

e) At what time does the ball hit the ground?

Ex. 6 An arch is to be built so that it is 6 feet wide at the base. Its shape can be represented by a parabola with the equation $y = -2x^2 + 12x$, where y is the height of the arch.

(a) Graph the parabola from $x = 0$ to $x = 6$.



(b) Determine the maximum height of the arch.

Solving Linear-Quadratic Systems Graphically

Intro to Geometry

AIM: → **To find the axis of symmetry using $x = \frac{-b}{2a}$**

→ ***To graphically solve Linear-Quadratic systems***

→ ***To identify the number of possible solutions for a Linear-Quadratic system***

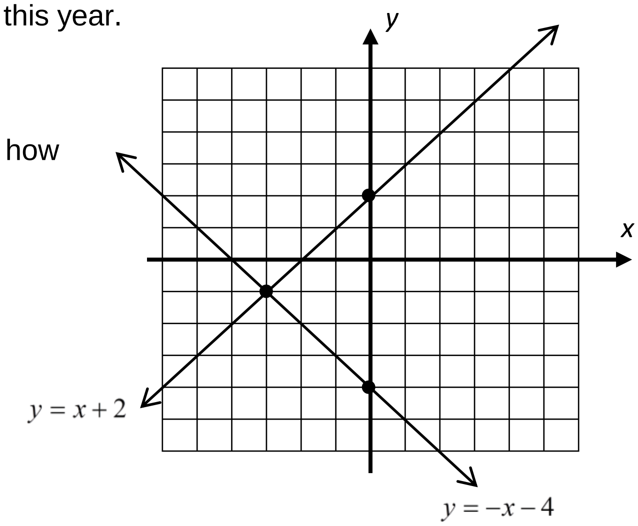
Recall: A ***system of equations*** is a set of two or more equations, such as:

$$y = x^2 - x + 1 \text{ and } y = 3x - 1$$

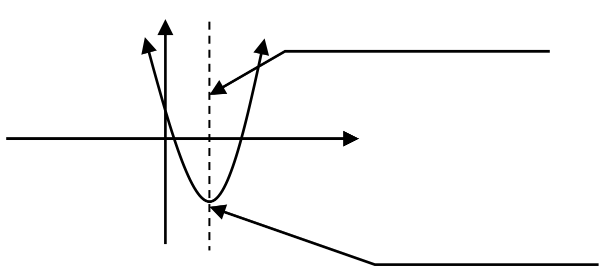
To ***solve a system of equations*** means to find all (x, y) pairs that satisfy ***both*** equations. You solved systems of linear equations earlier this year.

➡ When solving a system of linear equations, how did you identify the solution set?

➡ Write the solution set of this system below:



In this lesson, our focus will be to graphically find the solution set of a linear - quadratic system of equations.



The axis of symmetry and the turning point are essential features of a parabola that assist us in graphing a quadratic function of the form $y = ax^2 + bx + c$. We use them to help us generate a table of values.

Axis of Symmetry: $x = \frac{-b}{2a}$

Find the equation of the axis of symmetry for the following quadratic functions.

(a) $y = x^2 - 4x + 3$

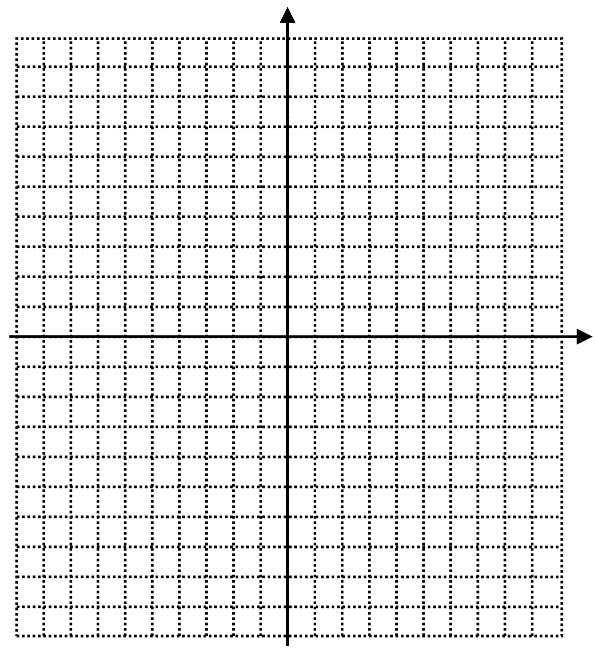
(b) $y = -x^2 + 2x + 6$

✷ How many solutions can a linear-quadratic system of equations have? Justify by drawing rough sketches in the space below.

<u>Ex. 1</u> Solve the following system of equations graphically and check your solutions.

$$y = x^2 - 6x$$
$$y = x - 6$$

(a) Find the axis of symmetry and the turning point.
Create a table of values for $y = x^2 - 6x$.



(b) Graph both equations and label.

(c) Find the intersection point(s) of the graphs.

(d) Check your solution(s) in both equations.

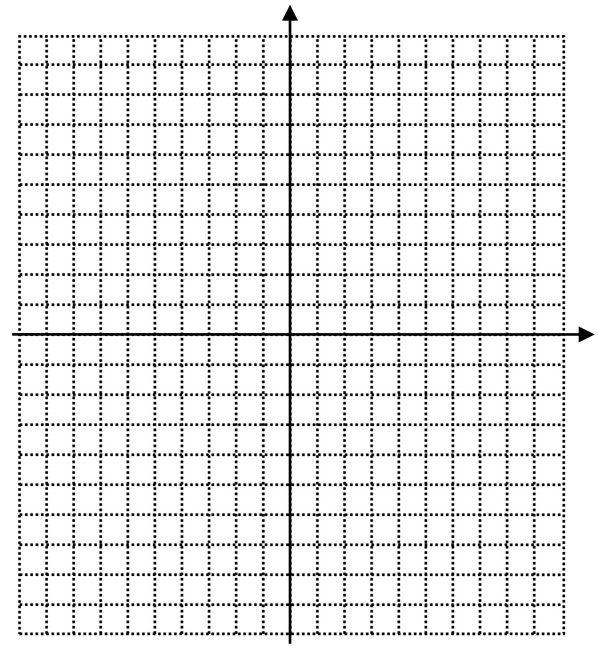
<u>Ex. 2</u> How many solutions does the following system of equations have?

$$y = 3x^2 + 2x + 5$$
$$y = -x - 2$$

(1) 1 (2) 2 (3) 3 (4) 0

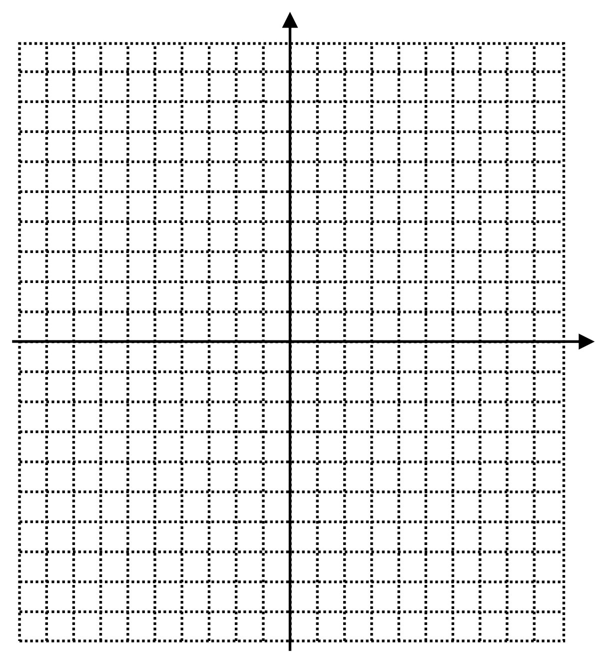
Ex. 3 Solve the following system of equations graphically and check your solutions.

$$y = x^2 - 4x + 3$$
$$y = -1$$



Ex. 4 Solve the following system of equations graphically and check your solutions.

$$y = x^2 + 3$$
$$y = \frac{1}{2}x - 4$$



Solving Linear-Quadratic Systems Graphically - HOMEWORK

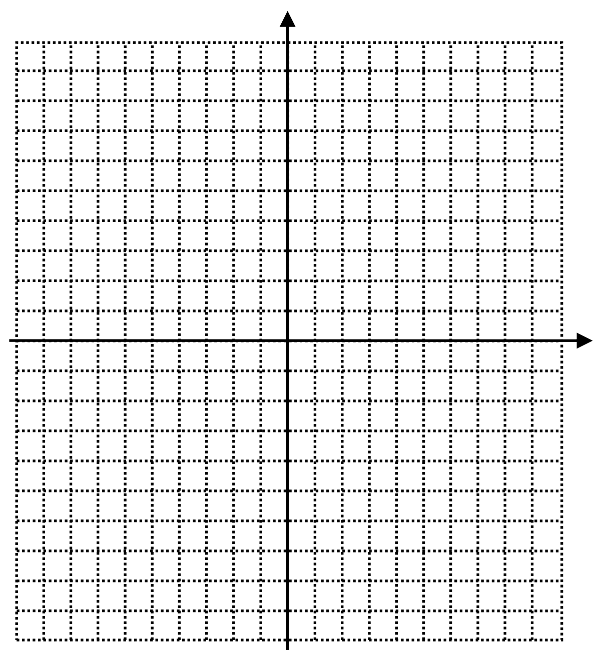
Intro to Geometry

1. Find the axis of symmetry (using $x = \frac{-b}{2a}$) and the turning point for each of the following quadratics.

(a) $x^2 - 2x + 15 = 0$ (b) $5x^2 + 20x + 2 = 0$ (c) $-x^2 + 11x - 18 = 0$

2. Solve the following Linear-Quadratic systems graphically and check your solutions.

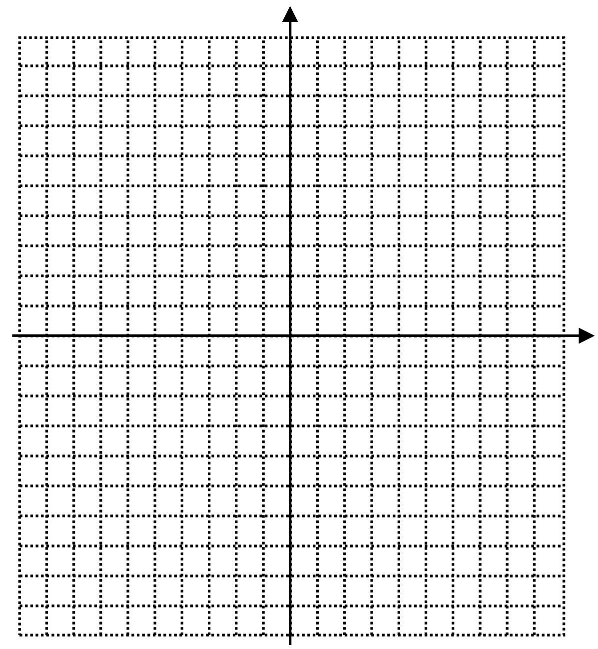
(a) $\begin{aligned} y &= x^2 + 4x + 5 \\ y &= 2x + 5 \end{aligned}$



(b)

$$y = 3x - 1$$

$$y = -x^2 + 4x + 1$$



UNIT 12

The 5 Fundamental Loci

Intro to Geometry

<u>AIM</u>: → To connect the definitions of points & lines in order to manipulate Starbursts & Twizzlers
→ To discover the five fundamental loci
→ To define the five fundamental loci using proper vocabulary

<u>Definitions</u>

1) Locus:

2) Line:

Starbursts represent ***<u>Points</u>***

Twizzlers represent ***<u>Lines</u>***

Locus 1: Follow the directions given below and answer the questions.

<u>Directions</u>

1) Place a point on your desk (in the center).
2) Place another point approximately 5 inches from the 1st point (anywhere on the desk).
3) Place another point approximately 5 inches from the 1st point (again, anywhere on the desk).
4) Continue putting points 5 inches from the original point (use all of the points that you have).

<u>Questions</u>

1) What figure did you create?

2) What do you think would happen if the 2nd, 3rd, 4th, etc. points were put 6 inches from the original point? 7 inches from the original point? 15 inches from the original point?

3) What vocabulary word can you use to describe the distance from the original point to any point on the circle?

4) What vocabulary word can you use to describe the 1st point put on the paper?

5) Remember—a locus is a set of points. Try writing the definition of a circle using the word *locus*.

Locus 2: Follow the directions given below and answer the questions.

Directions

1) Place two lines parallel to each other on your desk, approximately 8 inches apart (they can be vertical ***or*** horizontal lines).
2) Place a point so that it will be the same distance away from both lines.
3) Repeat Step 2 until you do not have any more points to use.
4) Remember—a line is made up of a series of points. Replace the points on your desk with a line.

Questions

1) What figure did you create?

2) What would happen to the locus if the original lines were put equally closer together?

3) What would happen to the locus if the original lines were put equally farther apart?

4) If you started this activity with two points 8 inches apart (see picture below) *instead of* two lines 8 inches apart, would you have created an identical locus? Explain why or why not.

Explanation:

Definitions

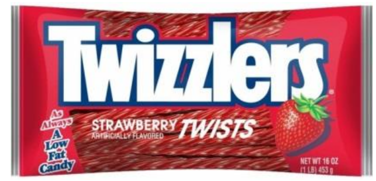
1) Equidistant:

2) Bisect:

3) Perpendicular Bisector:

Starbursts represent ***Points***

Twizzlers represent ***Lines***



Locus 3: Follow the directions given below and answer the questions.

Directions

1) Place a point anywhere on your desk.
2) Place another point approximately 5 inches from the 1st point (anywhere on the desk).
3) Place a new point midway between the other points.
4) Continue placing points midway between the original 2 points.

Questions

1) What figure did you create?

2) How do you know that all of the points creating the locus are midway between the 2 original points?

3) Using the words **segment** and **perpendicular bisector**, try to finish the definition for the figure you created.

The locus of points is the…

Locus 4: Follow the directions given below and answer the questions.

Directions

1) Place one line on your desk.
2) Place a point so that it will be approximately 8 inches away from the line.
3) Repeat Step 2 until you do not have any more points to use.
4) Remember—a line is made up of a series of points. Replace the points on your desk with a line.

Questions

1) What figure did you create?

2) At what point did you realize that locus was 2 lines?

3) What are some mathematical vocabulary words that can be used to create the definition for this locus?

Locus 5: Follow the directions given below and answer the questions.

Directions

1) Place 2 intersecting lines on your desk.
2) This may get confusing, so have sharp eyes…put a point equidistant from these intersecting lines (do one locus at a time).
3) Continue creating the loci (there are 2 loci in this problem) by putting points equidistant from the intersecting lines.

Questions

1) What figures did you create?

2) How many loci did you create?

3) Is there anything you could have done to make the same loci, but in a less confusing way?

4) Using the words **lines**, **bisect**, and **angles**, try to finish the definition for this locus.

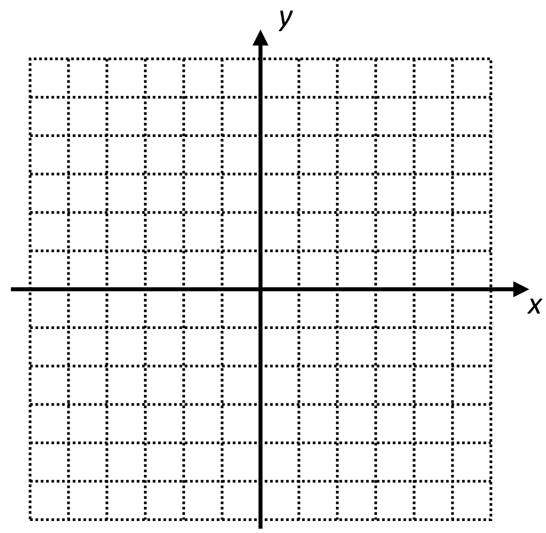
The locus of points is…

Locus & Coordinate Geometry
Intro to Geometry

<u>AIM</u>: → *To draw a locus of points given various conditions*
→ *To state the equation of a locus*

Recall: A locus is a collection of points that meet a given condition.

<u>Ex. 1</u> Draw the locus of all points 4 units from the point $(0,0)$. State the equation of this locus.



Theorem: The locus of all points a fixed distance from a given point is

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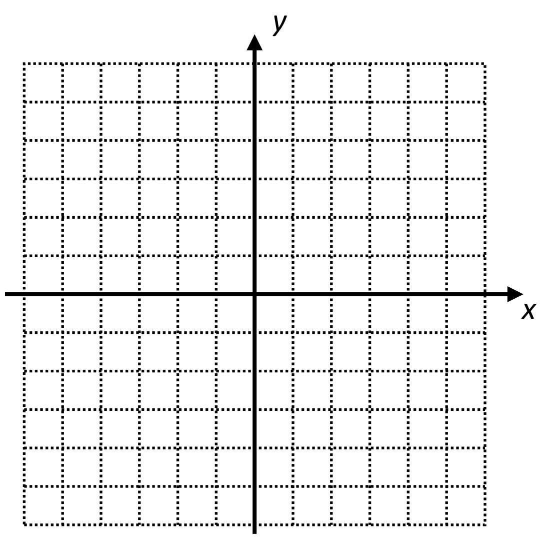
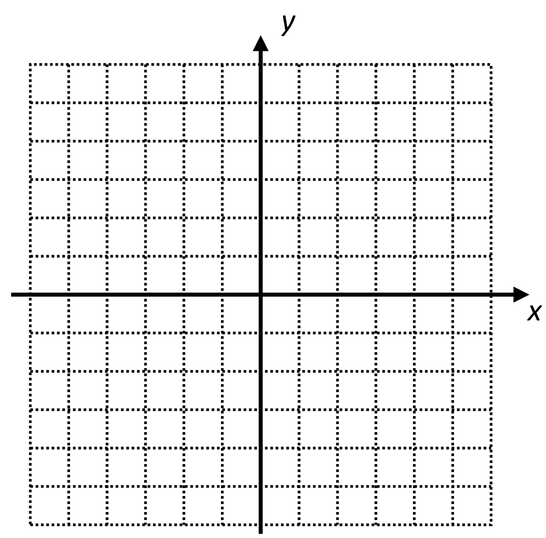
<u>Ex. 2</u> Graph the line $y=2$ and the locus locus (collection) of points at a distance of distance of 3 units from the line.

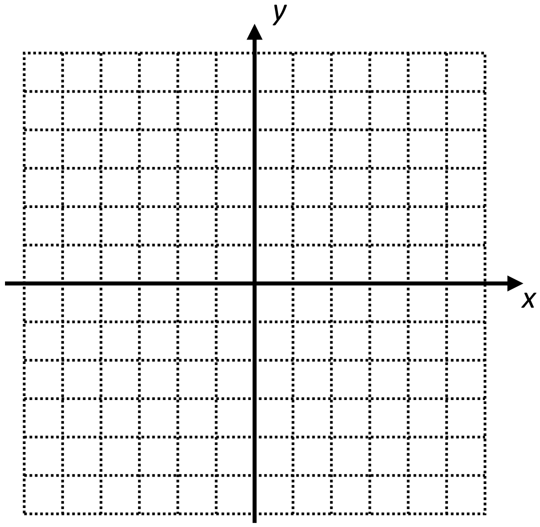


<u>Ex. 3</u> Draw the locus of points 2 units from the line $x=-1$ and write the equations for the locus.

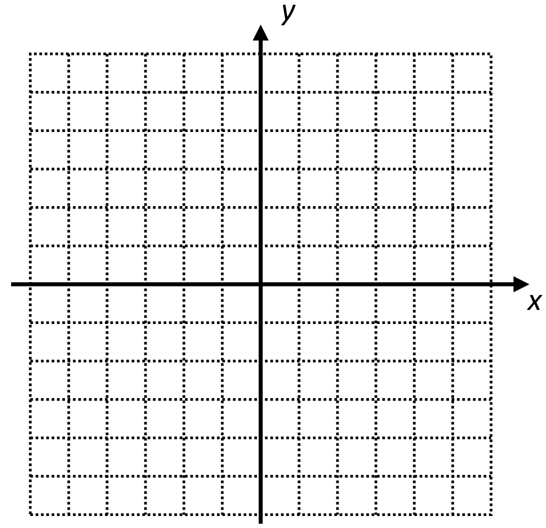


Theorem: The locus (collection) of points a fixed distance from a given line is ____________________

<u>Ex. 4</u> Draw the locus ***equidistant*** (the same distance) from the lines $y=-1$ and $y=5$. Write the equation of the locus.

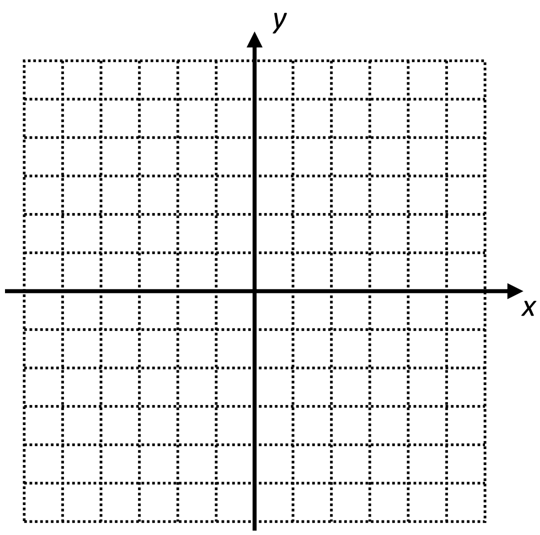


<u>Ex. 5</u> Draw the locus of points equidistant from the lines $x=-4$ and $x=2$. Write the equation of the locus.

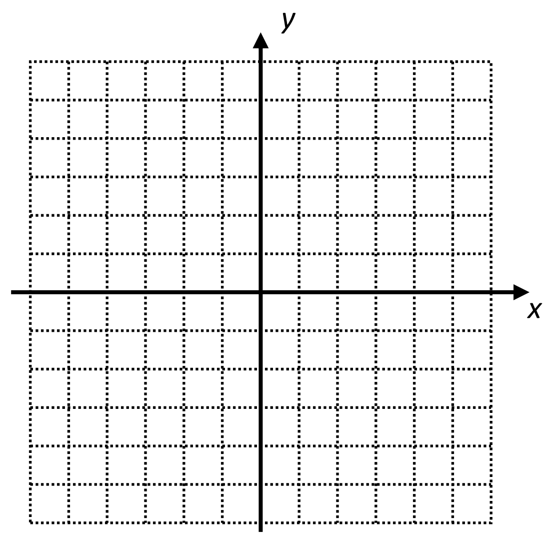


Theorem: The locus of points equidistant (the same distance) from two parallel lines is

<u>Ex. 6</u> On the same set of axes, draw the Locus of points equidistant from the lines $y=-\frac{3}{2}x+5$ and $y=-\frac{3}{2}x-1$. Write the equation of the locus.



<u>Ex. 7</u> Graph the points $(-1,3)$ and $(5,3)$. Graph the locus of points equidistant from these two points. Write the equation of the locus.



Theorem: The locus of points equidistant from two points is

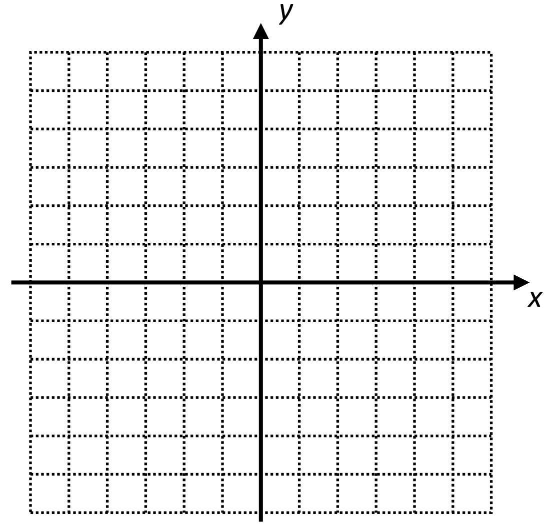
Ex. 8 Which of the following equations represents the locus of all points equidistant from $(2,4)$ and $(2,-2)$?

(1) $x = 1$

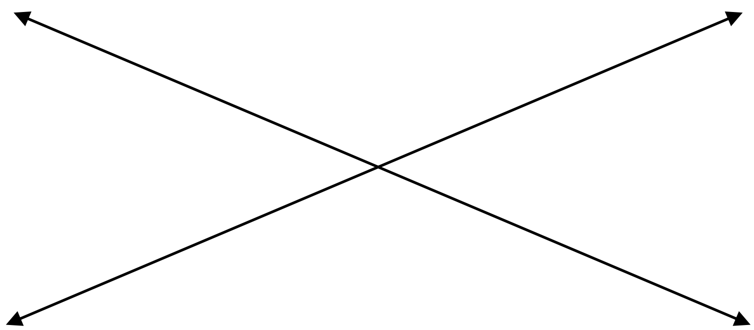
(2) $y = 1$

(3) $x = 2$

(4) $y = 2$



Ex. 9 Given the following two intersecting lines, draw the locus of all points equidistant from the two lines.



Theorem: The locus of points equidistant from two intersecting lines is

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Locus & Coordinate Geometry - HOMEWORK
Intro to Geometry

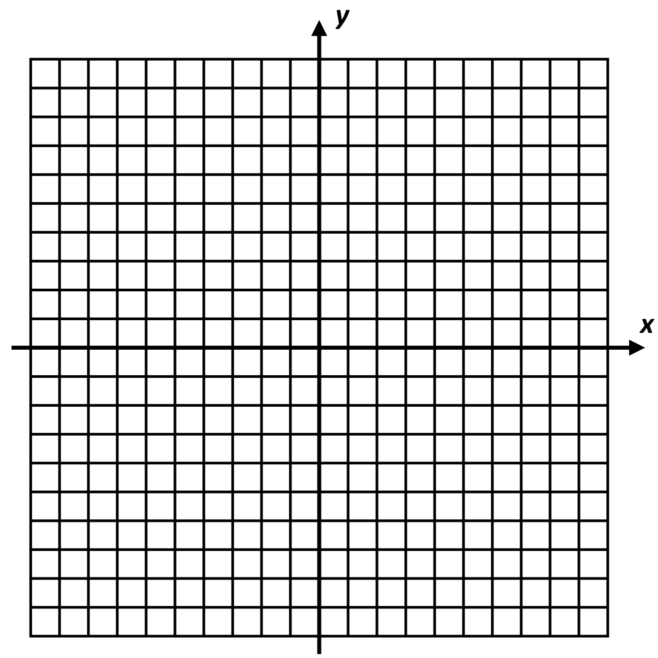
In problems 1 – 4, write the ***equation*** of the locus of points equidistant from the given points. Use graph paper to help find the equations.

1. $(-2,2)$ and $(6,2)$

3. $(-4,5)$ and $(0,5)$

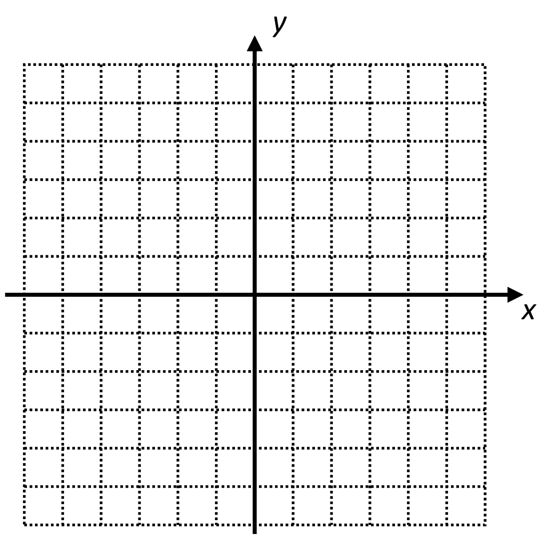
2. $(-8,-4)$ and $(-2,-4)$

4. $(8,2)$ and $(8,0)$

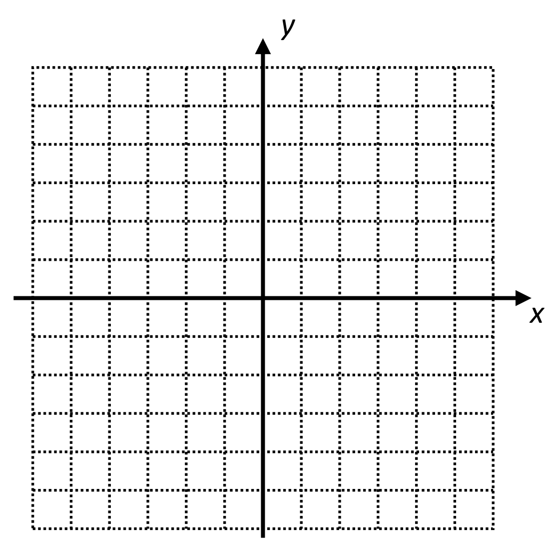


5. Draw the locus of points satisfying the given conditions and ***write the equation*** for the locus.

3 units from the line $y=1$

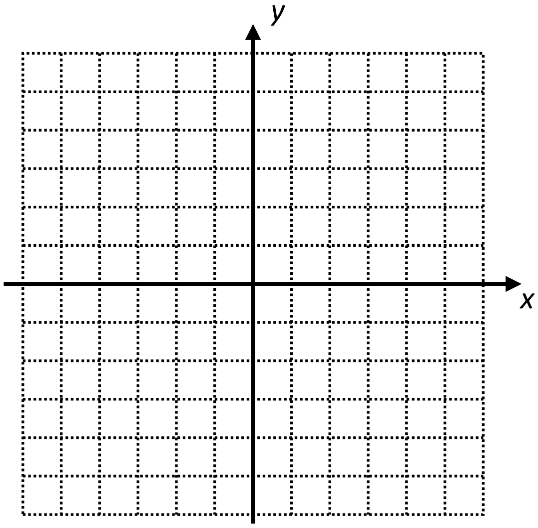


2 units from the line $x=3$

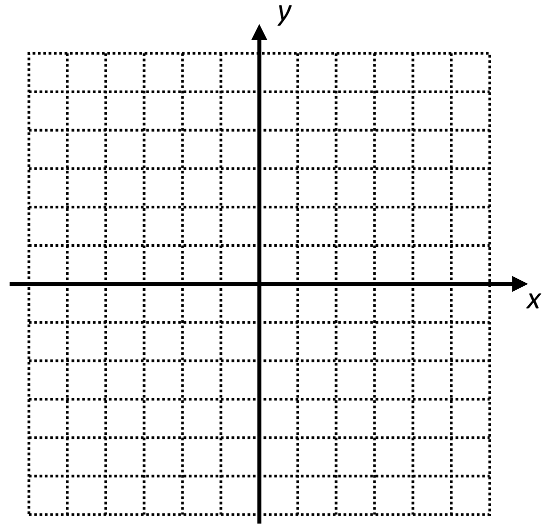


6. Draw the locus of points satisfying the given conditions and ***write the equation*** for the locus.

Equidistant from the lines $y=1$ and $y=5$



Equidistant from the lines $x=-5$ and $x=1$



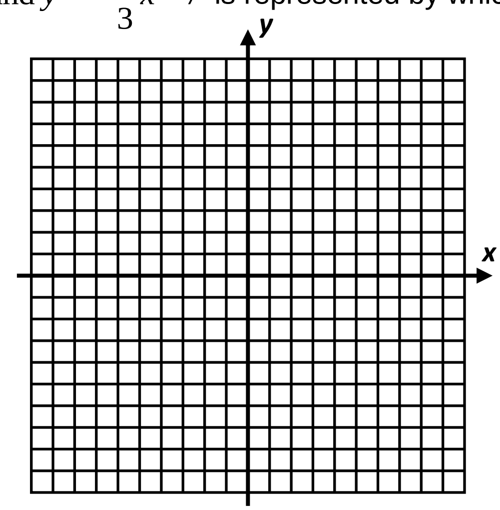
7. All points equidistant from the lines $y=-\frac{2}{3}x+5$ and $y=-\frac{2}{3}x-7$ is represented by which of the following equations?

(1) $y=-\frac{2}{3}x+2$

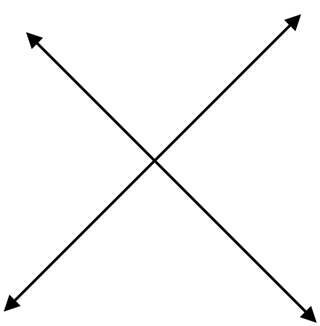
(2) $y=-\frac{2}{3}x-2$

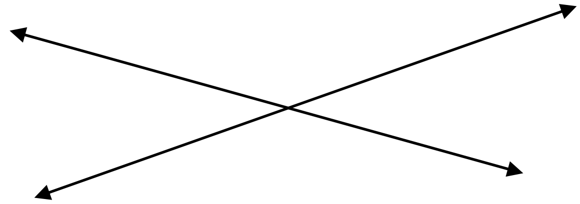
(3) $y=\frac{3}{2}x-1$

(4) $y=-\frac{2}{3}x-1$



8. Sketch the locus of all points equidistant from the following pairs of intersecting lines.





Multiple Loci Conditions
Intro to Geometry

AIM: → *To graph and solve compound loci in the coordinate plane*

Do Now

Which of the equations below describes the locus of all points 5 units from the point (-3, 4)?

(1) $(x-3)^2+(y+4)^2=25$ (3) $(x+3)^2+(y-4)^2=5$

(2) $(x+3)^2+(y-4)^2=25$ (4) $(x-3)^2+(y+4)^2=5$

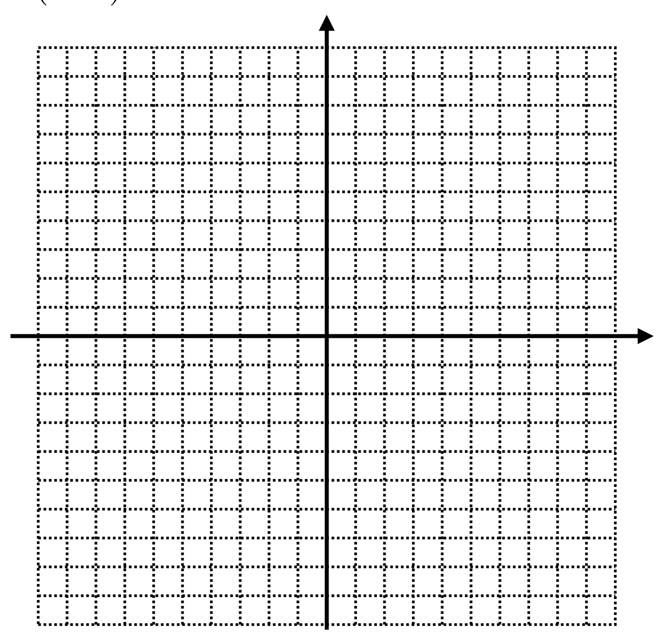
When we work with multiple loci problems, we are trying to find where loci conditions overlap. This is similar to when we solve a system of equations graphically.

Ex. 1 What is the ***total number*** of points 5 units from $(-3,4)$ *and* 2 units from the x-axis?

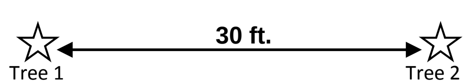
(a) Plot the first locus condition in dashed lines.

(b) Ignore the first locus and plot the second locus condition in dashed lines.

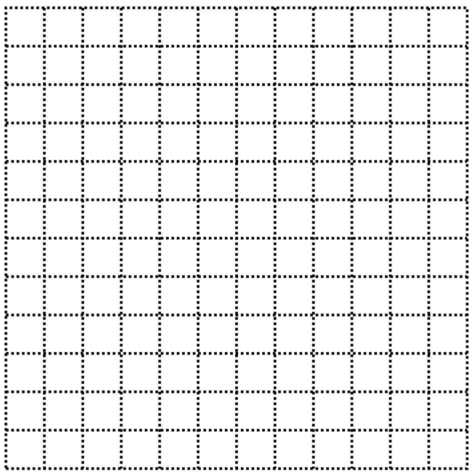
(c) Draw points at where the two overlap and count the number of points.



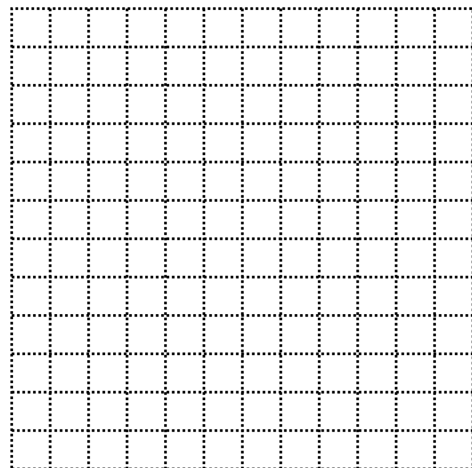
Ex. 2 Lyra's backyard has two trees that are 30 feet apart, as shown in the accompanying diagram. She wants to place lampposts so that the posts are 20 feet from both of the trees. Draw a sketch to show where the lampposts could be placed in relation to the trees. How many locations for the lampposts are possible? *The diagram shows a bird's eye view of the trees.*



Ex. 3 Points C and D are 4 units apart. What is the total number of points 3 units from D and equidistant from C and D?



Ex. 4 Lines *a* and *b* intersect at *E*. What is the total number of points 3 units from *E* and equidistant from lines *a* and *b*?



Ex. 5 Alex told Christopher that he has a treasure map that indicates the location of a family time capsule. It is supposed to be found exactly 200 yards from the birdbath and 700 yards from Eagle Road. If the birdbath is 500 yards from Eagle Road, can the boys locate the time capsule with this information? If so, in how many places could it be found?

Ex. 6 Point *P* is located on line $\overleftrightarrow{AB}$. Describe the locus of points that are:

(a) 5 units from the line $\overleftrightarrow{AB}$

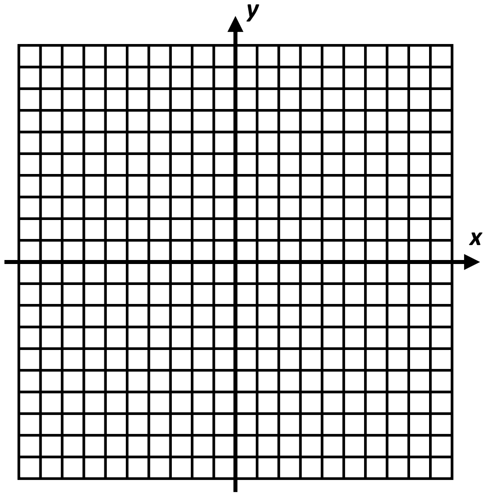
(b) 5 units from point *P*

How many points satisfy both conditions above?

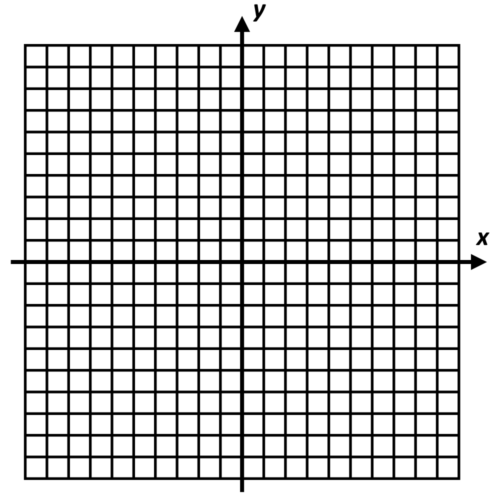
Multiple Loci Conditions - HOMEWORK
Intro to Geometry

1. Give the total number of points in the locus.

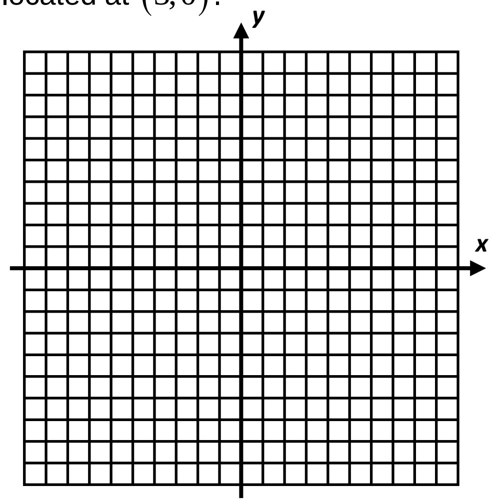
(a) Locus of 2 units from (-6, 4) and 5 units from the *y*-axis.



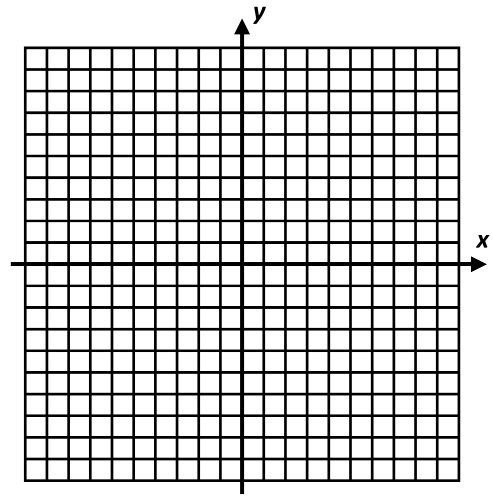
(b) Locus of 2 units from the origin and 1 unit from the line $x = 3$.



(c) Locus of 2 units from the origin and 4 units from point *Q* if point *Q* is located at $(3,0)$.



(d) Locus of points 5 units from $P(3,2)$ and equidistant from the *x* and *y*-axes.



2. A tree is located 30 feet east of a fence that runs north and south. Olivia tells her brother Dante that their dog buried his hat a distance of 15 feet from the fence and about 20 feet from the tree. Draw a sketch to show where Dante should dig to find his hat. How many locations are possible?

Surface Area of Prisms, Spheres, and Cylinders

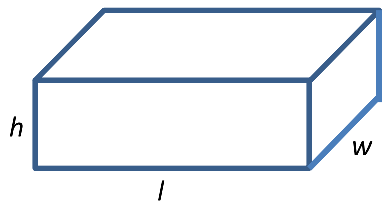
Intro to Geometry

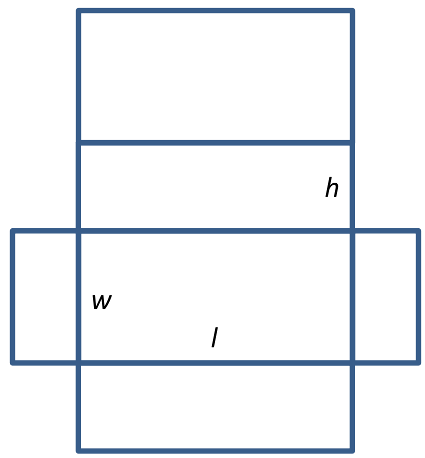
<u>Definition</u>: The **surface area** of a polyhedron is the sum of the areas of the faces.

<u>Surface Area of a Cylinder</u>

To find the surface area of a prism, we can deconstruct a prism, creating *net* to represent the sides.

Exercise # 1: Find the surface area of the rectangular prism below in terms of *l*, *w*, and *h*.





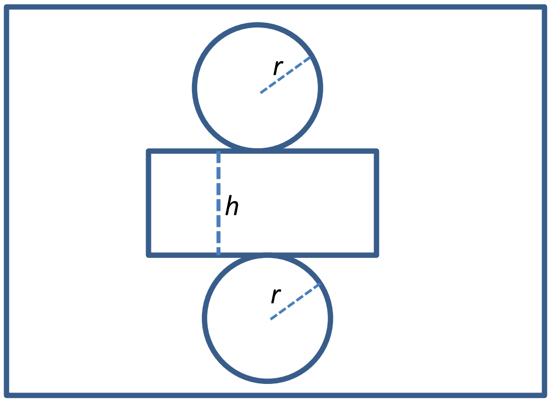
FORMULA FOR SURFACE AREA OF A (rectangular)PRISM: SA = ___________________________________

Exercise # 2: Find the surface area of a rectangular solid (prism)with the dimensions 5 cm by 8 cm by 3 cm.

Exercise # 3: The surface area of a cube is $54\ in^2$. Find the dimensions of the cube.

Finding the Total Surface Area of a Cylinder

To find the total surface area of a cylinder, we have to find the area of the section connecting the two bases and add the sum of the area of the two bases, which are circles. If you look at the picture below, you'll see the center part connecting the two bases is actually a rectangle, where the height (h) and the circumference of the base ($2\pi r$) are the length and width of the rectangle. So:



Total Surface Area of a Cylinder

SA = $\boldsymbol{2\pi r^2 + 2\pi rh}$

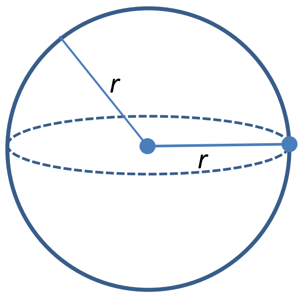
Exercise # 4: Find the total surface area of a right circular cylinder that has a height of 12 inches and a *diameter* of 10 inches.

Exercise # 5: Find the surface area of a cylinder when $r = 3.5\ cm$ and $h = 16\ cm$.

Exercise # 6: Find the surface area of a cylinder when $r = 2\ in.$ and $h = 4\ in.$

Surface Area of a Sphere

Because the 3 dimensional share of a sphere has no straight edges, the surface area is complex, although not that different from the area of a circle.



Surface area of a Sphere

SA= $4\pi r^2$

Exercise # 7: Find the surface area of the sphere with the given radius. Leave your answer in terms of π .

a.) $r = 3$ *in.* b.) $r = 1.5$ *in.* c.) $r = 1.6$ *in.*

Exercise # 8: Find the surface area of the sphere with the given radius. Round your answer to the *nearest hundredth*.

a.) $r = 5$ *in.* b.) $r = 2.4$ *in.* c.) $r = 2\pi$ *in.*

Surface Area of Prisms, Spheres, and Cylinders

Intro to Geometry Homework

1. Find the surface area of a rectangular prism whose length is 2ft, width is 4ft, and height is 3 ft.

2. Find the surface area of a rectangular prism whose length is 6.8 in, width is 3.2 in, and height is 5.5 in.

3. Find the surface area *to the nearest hundredth* of a cylinder whose radius is 4 m and height is 6 m.

4. Find the surface area *in terms of* π , of a cylinder whose radius is 3 in. and height is 11 in.

5. Find the surface area *to the nearest hundredth* of a cylinder whose diameter is 12 ft and height is 8 ft.

6. Find the surface area *in terms of* π , of a sphere whose radius is 10 in.

7. Find the surface area *to the nearest tenth,* of a sphere whose radius is 2.3 yd.

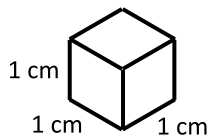
8. Find the surface area *in terms of* π , of a sphere whose diameter is 14 ft.

Volume
Introduction to Geometry

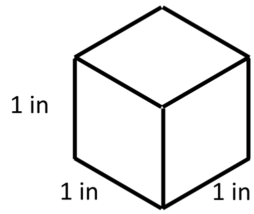
Volume is an important measurement in mathematics because it quantifies (assigns a number to) the amount of three-dimensional space an object occupies. It is the 3-D equivalent of area in 2-D and perimeter in 1-D. We begin with a review problem from Algebra.

***Exercise* #1:** What is the volume of each of the following cubes?

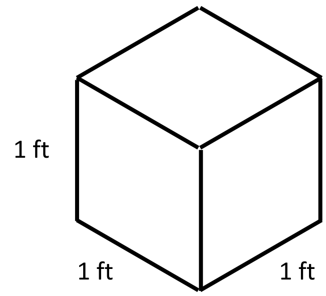
(a)



Volume =



Volume =

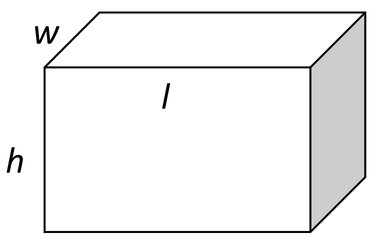


Volume =

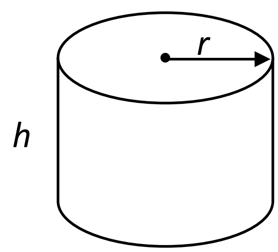
<u>The Rectangular Prism (Box) and the Right Circular Cylinder (Soda Can)</u> - These two common shapes have volume formulas that are easy to derive. In the next exercise you will come up with these formulas.

***Exercise* #2:** Find the volume formulas for the rectangular prism and right circular cylinder.

(a)

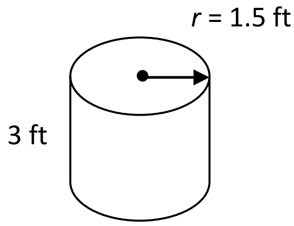


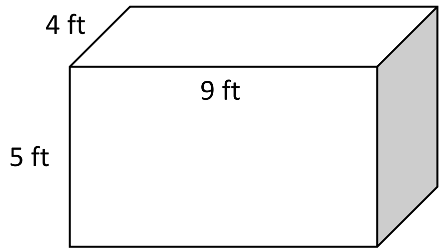
(b)



Volume of a Rectangular Prism: ______________________________

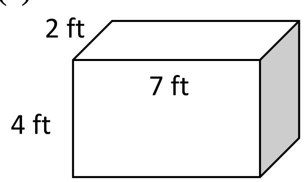
***Exercise* #3:** Find the volume of each object.



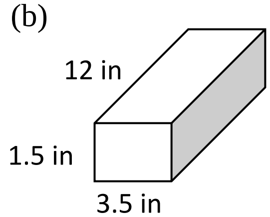


Exercise # 4: Calculate the volume of each of the following rectangular prisms. Make sure to include units.

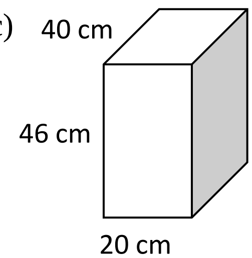
(a)



(b)

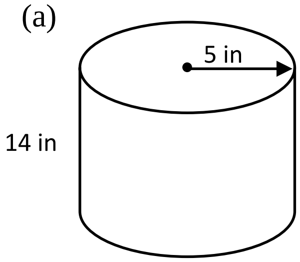


(c)

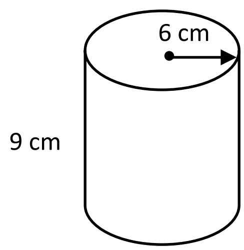


Exercise # 5 Calculate the volume of each of the following right circular cylinders. Express your answers rounded to the nearest *tenth*. Make sure to include units.

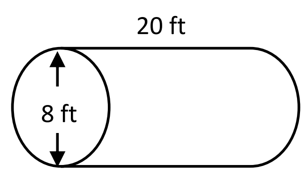
(a)



(b)



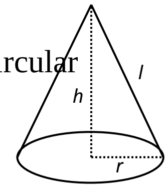
(c)



Watch Out! The diameter is given.

<u>Cone:</u> a **cone** is a three-dimensional shape that tapers smoothly from a flat, round *base* to a point called the *vertex*.

<u>Fact</u> : It takes 3 cones to fill a cylinder with the same height and same circular base. Therefore,



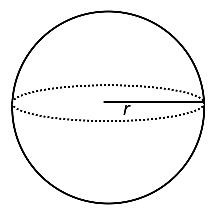
<u>Volume of a Cone</u> : $V = \frac{\pi r^2 h}{3}$

Exercise # 5: The height of a cone is 24 cm and the radius of the base measures 10 cm. Find the volume to the *nearest tenth*.

Exercise # 6 : The height of a cone is 15 in and the diamete4r is 16 in. Find the volume to the *nearest cubic inch*.

<u>Sphere</u>: a **sphere** is the set of all three dimensional points a fixed distance, *radius*, from a given point, or **center**.

<u>Volume of a Sphere</u> : $V = \frac{4}{3}\pi r^3$



Exercise # 7: Find the volume of the sphere with the given radius. Leave your answers in terms of π .

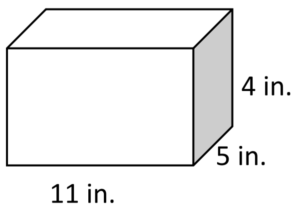
a.) r = 3 in   b.) r = 1.5 in   c.) r = 12 m

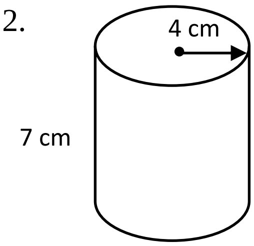
Exercise # 8: Find the volume of the sphere with the given radius. Round your answers to the *nearest hundredth*.

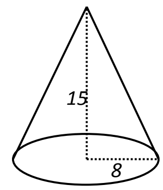
a.) r = 5 in   b.) r = 2.4 m   c.) r = 5.3 ft

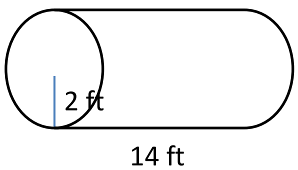
Volume
Introduction to Geometry Homework

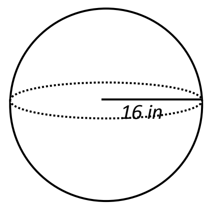
Find the volume of the given shapes. Round to the nearest tenth where necessary.

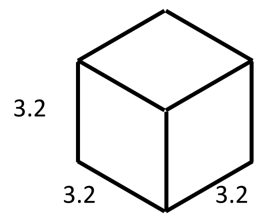
1. 

2. 

3. 

4. 

5. 

6. 

7. The height of a cone is 8 cm and the radius of the base measures 6 cm. Find the volume to the *nearest tenth*.

8. Find the volume of the sphere with a radius of 12 cm. Round your answer to the *nearest hundredth*.

9. Find the volume of a sphere whose diameter is 9 ft. Leave your answer in terms of π