Name: _____ **UNIT 7 BOOK**

Quadrilateral Family Tree

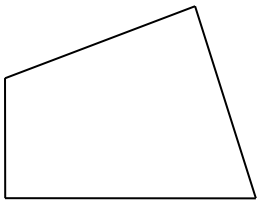
Intro to Geometry

-"Turn and talk" – take a few minutes to come up with as many properties that make these shapes unique. Write those properties inside the figures.
Under the name of each shape, we will write the properties of their diagonals.

Quadrilaterals & Properties of Parallelograms

AIM: → *To define quadrilateral*
→ *To define parallelogram and state its special properties*

Quadrilateral : A polygon with 4 sides.

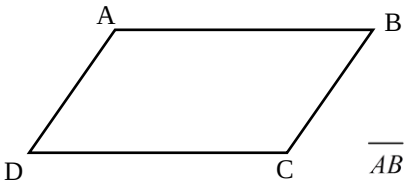


➡ Notice that the 4 sides do not have to be congruent for the figure to be a quadrilateral

There are special quadrilaterals that have distinct properties. One special quadrilateral is a **parallelogram**.

Parallelogram : A quadrilateral with ***two*** pairs of ***parallel*** sides.

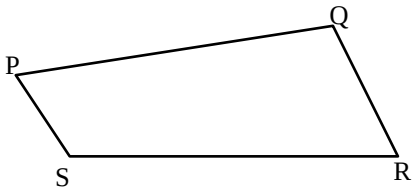
Example of a Figure that ***IS*** a Parallelogram



$\overline{AB} \parallel \overline{DC}$

$\overline{AD} \parallel \overline{BC}$

Example of a Figure that ***IS NOT*** a Parallelogram



Important Properties that Follow from the Parallel Lines of a Parallelogram

1. Opposite angles of a parallelogram are congruent (equal in measure).

2. Consecutive angles of a parallelogram are supplementary (sum to 180^o).

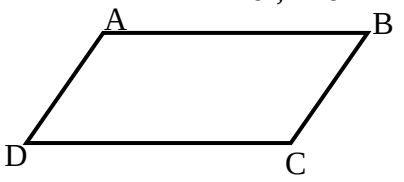
3. Opposite sides of a parallelogram are congruent (equal in measure).

4. Diagonals of a parallelogram bisect each other (have the same midpoint).

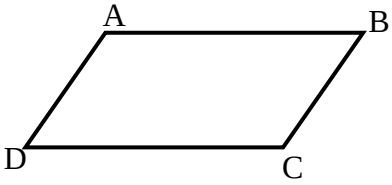
Practice with Parallelogram Properties

Together

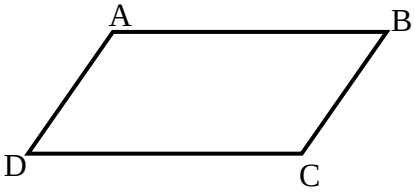
1. $m\angle A = 110^\circ$, find $m\angle C$



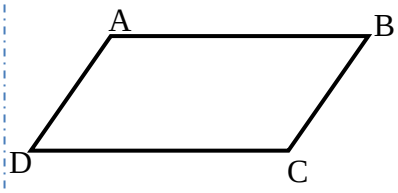
2. $m\angle D = 45^\circ$ find $m\angle A$.



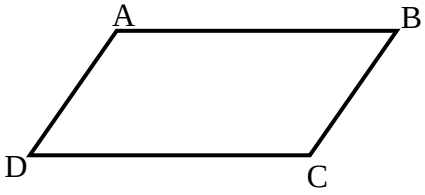
3. $m\angle D = 3x - 10$ and $m\angle B = x + 20$, find x.



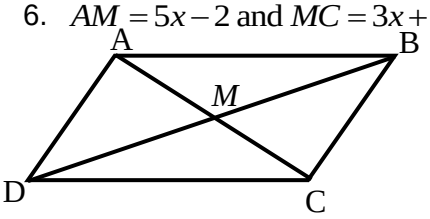
4. $m\angle A = 2x + 10$ and $m\angle B = 3x - 30$, find x.



5. $AD = 3x + 8$ and $BC = x + 14$, find BC.



6. $AM = 5x - 2$ and $MC = 3x + 28$, find AC.



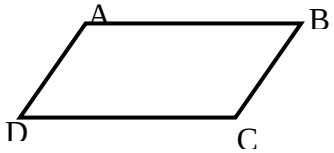
(7- 8) Solve each of the following problems. First, draw a diagram and properly label it.

7. In parallelogram PQRS, $m\angle P = x + 15$ and $m\angle Q = 5x - 15$, find $m\angle R$.

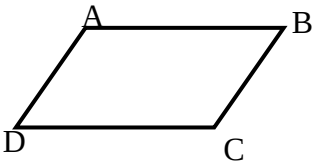
8. In parallelogram WXYZ, $WX = 7x - 4$, $XY = 6x - 2$, and $YZ = 4x + 11$, find XY.

Independent Practice

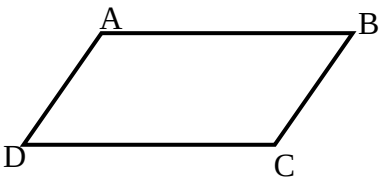
1. $m\angle D = 48^\circ$, find $m\angle B$



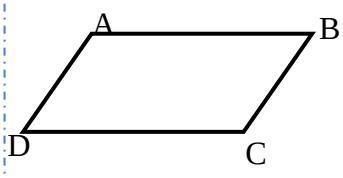
2. $m\angle C = 120^\circ$ find $m\angle B$



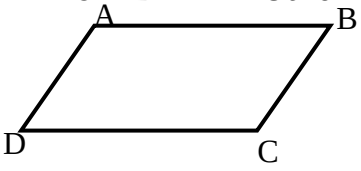
3. $m\angle A = 8x - 2$ and $m\angle C = 5x + 46$, find x.



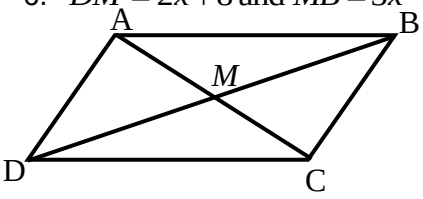
4. $m\angle A = 6x + 10$ and $m\angle D = 4x - 20$, find x.



5. $AD = 4x + 18$ and $BC = 3x + 34$, find BC.



6. $DM = 2x + 8$ and $MB = 3x - 7$, find BD.



(7- 8) Solve each of the following problems. First, draw a diagram and properly label it.

7. In parallelogram JKLM, $m\angle K = 7x + 23$ and $m\angle M = 5x + 31$, find $m\angle J$.

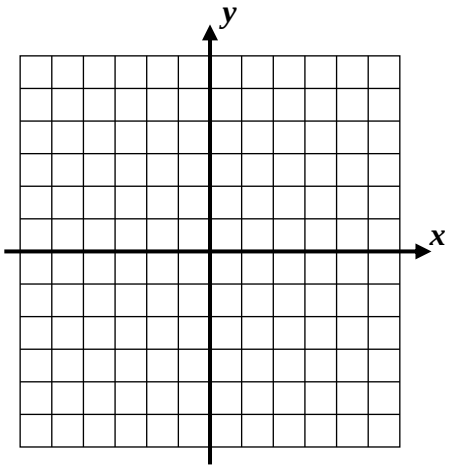
8. In parallelogram EFGH, $EF = 5x + 14$, $FG = 3x + 2$, and $GH = 4x + 23$, find HE.

Proving Figures are Parallelograms
Intro to Geometry

AIM: → To Prove a Quadrilateral is a Parallelogram using the diagonals of the quadrilateral.

Example: Quadrilateral ABCD has vertices A(2, 5), B(3, 2), C(-1, -1) and D(-2, 2). Prove using coordinate geometry that ABCD is a parallelogram.

Step 1: Plot the quadrilateral on graph paper.



Step 2: Find the midpoint of each diagonal of the quadrilateral.

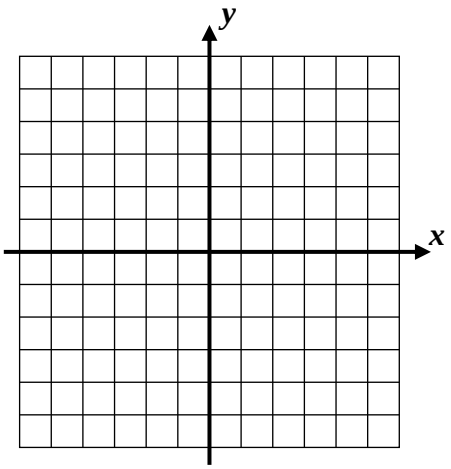
$\overline{AC}$

$\overline{BD}$

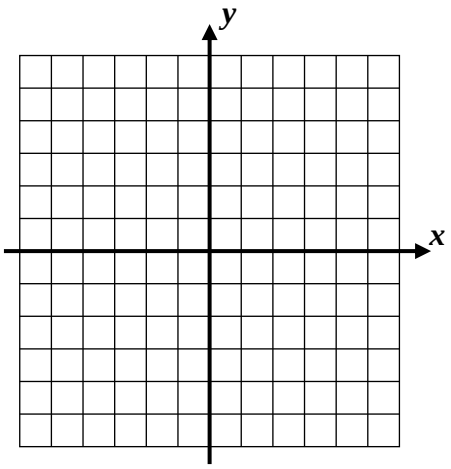
Step 3: Determine whether or not the diagonals bisect each other (and state why).

Step 4: Write a conclusion.

1. Quadrilateral PQRS has vertices P(5, 2), Q(2, -2), R(-3, -2) and S(0, 2). Prove using coordinate geometry that PQRS is a parallelogram.



2. Carla thinks that ABCD, which has vertices of A(1, 4), B(4, 1), C(-1, -2), and D(-5, 0), is a parallelogram. Prove using coordinate geometry that Carla is *wrong*.

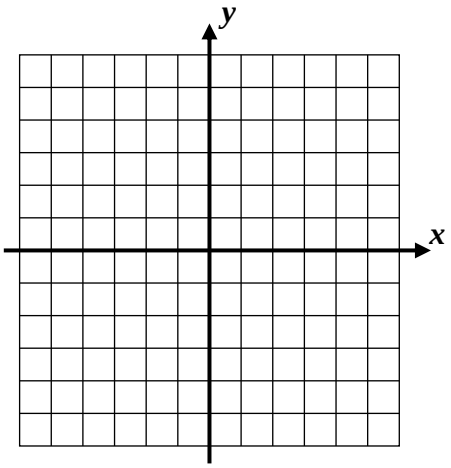


Proving Figures are Parallelograms - Homework
Intro to Geometry

In each of the following problems, use the method we outlined in class to prove that the quadrilateral is a parallelogram.

1. Quadrilateral ABCD has vertices A(3, 1), B(-1, -1), C(-6, 2) and D(-2, 4). Prove using coordinate geometry that ABCD is a parallelogram.

Step 1: Plot the quadrilateral on the axes shown below.



Step 2: Find the midpoint of each diagonal of the quadrilateral.

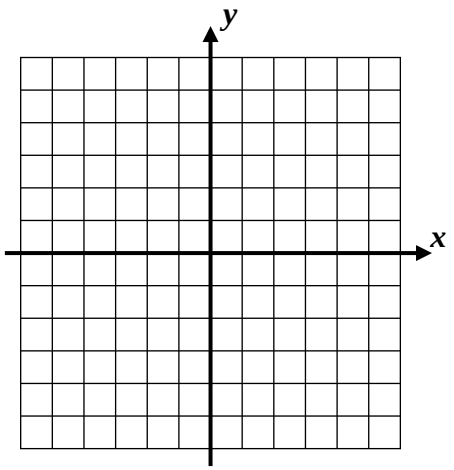
$\overline{AC}$

$\overline{BD}$

Step 3: Determine whether or not the diagonals bisect each other (and state why)

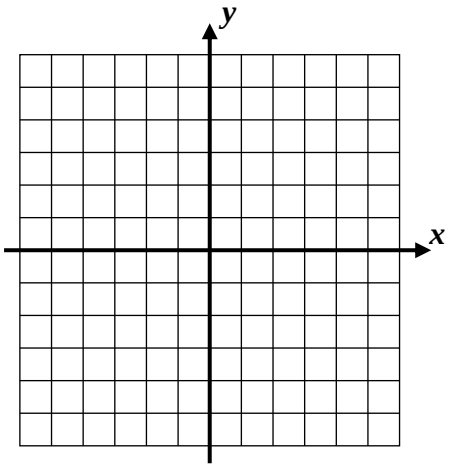
Step 4: Write a conclusion.

2. Quadrilateral PQRS has vertices P(5, 2), Q(2, -2), R(-2, -2) and S(-3, 2). Prove using coordinate geometry that PQRS ***is not*** a parallelogram.



Question: Although PQRS is not a parallelogram, it is a special type of quadrilateral. What is its name? {You will need to think back to previous math courses}

3. Quadrilateral WXYZ has vertices W(1, 5), X(6, 0), Y(3, -3) and Z(-2, 2). Prove using coordinate geometry that WXYZ is a parallelogram.



Proving Figures are Rectangles
Intro to Geometry

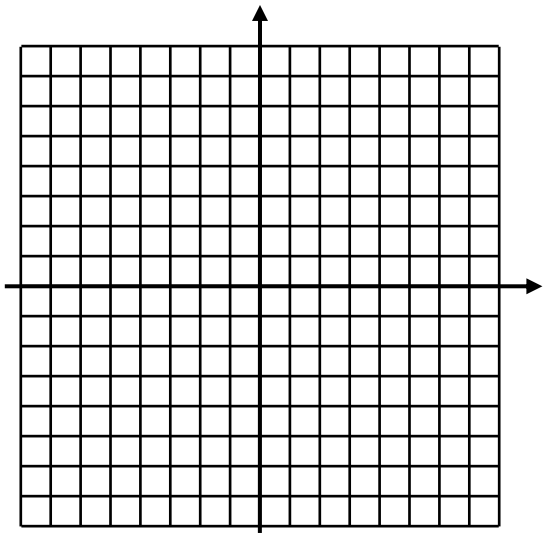
AIM: → *To Prove a Quadrilateral is a Rectangle using the diagonals of the quadrilateral.*

To Prove that a Figure is a Rectangle

✶Prove that the quadrilateral is a Parallelogram that has CONGRUENT DIAGONALS.

Ex. 1 Quadrilateral *ABCD* has vertices $A(2,5)$, $B(4,2)$, $C(-2,-2)$, and $D(-4,1)$. Prove using coordinate geometry that *ABCD* is a rectangle.

Step 1: Plot and label the quadrilateral on the grid.



Step 2: Prove ABCD is a parallelogram (use midpoints).

$\overline{AC}$:

$\overline{BD}$:

Step 3: Calculate the DISTANCE of each diagonal and make a conclusion based upon their distances.

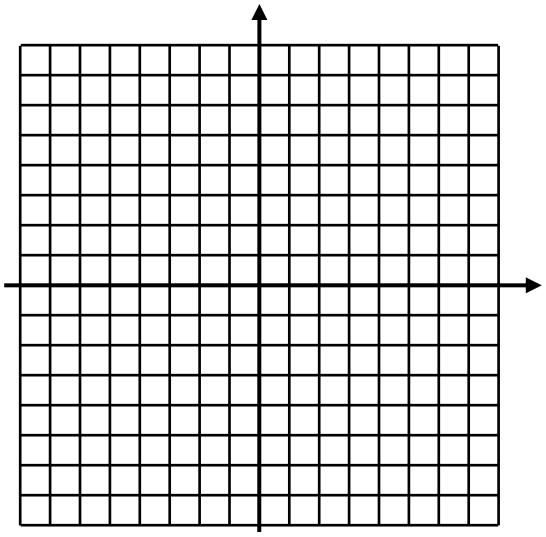
$\overline{AC}$:

$\overline{BD}$:

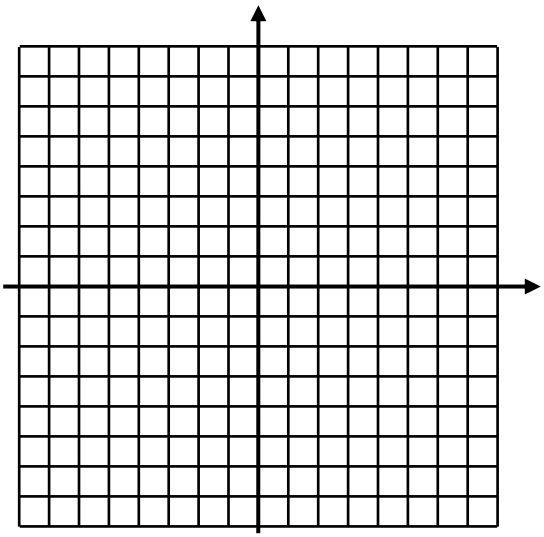
Step 4: Write a conclusion about the quadrilateral.

ABCD is a rectangle because it is a parallelogram with _____ diagonals.

Ex. 2 . Quadrilateral *PQRS* has vertices *P*(2, 4), *Q*(3, 1), *R*(-3, -1) and *S*(-4, 2). Prove using coordinate geometry that *PQRS* is a rectangle.

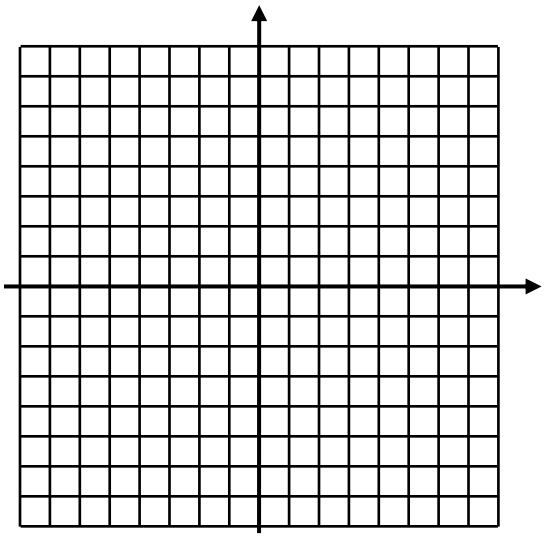


Ex. 3 Quadrilateral *ABCD* has vertices *A*(2, 4), *B*(1, -1), *C*(-4, -2), and *D*(-3, 3). Prove using coordinate geometry that *ABCD* is ***NOT*** a rectangle.



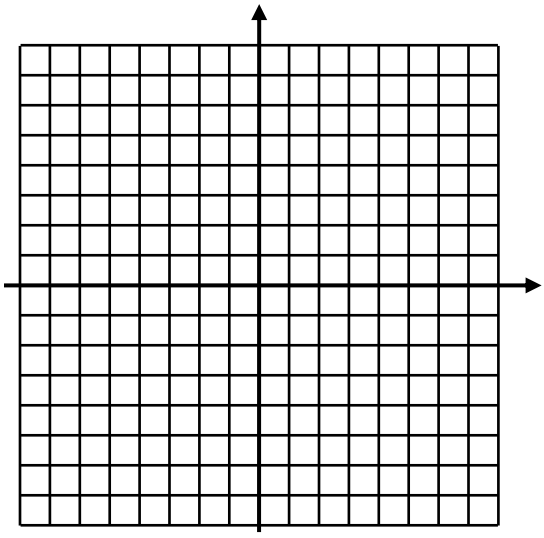
Proving Figures are Rectangles - Homework
Intro to Geometry

1. Quadrilateral MNPQ has vertices M(-2, 5), N(2, -3), P(-2, -5) and Q(-6, 3). Prove using coordinate geometry that MNPQ is a rectangle.



2. Quadrilateral MNPQ has vertices M(-3, 4), N(3, 0), P(1, -4) and Q(-5, 0). Prove using coordinate geometry that:

(a) MNPQ is a parallelogram.



(b) MNPQ is *not* a rectangle.

Proving Figures are Rhombuses (Rhombii)
Intro to Geometry

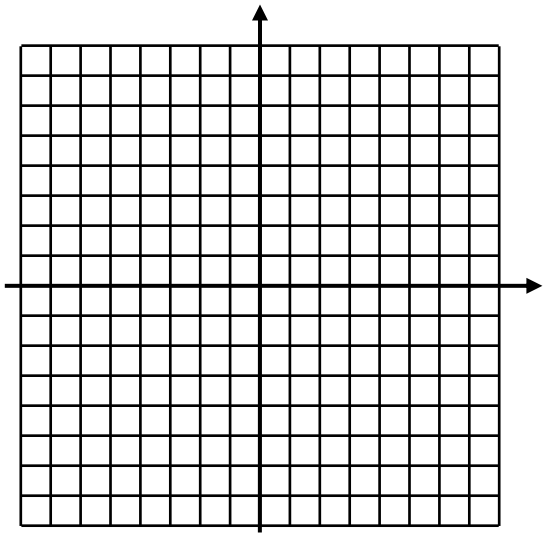
AIM: → *To Prove a Quadrilateral is a Rhombus using the diagonals of the quadrilateral.*

To Prove that a Figure is a Rhombus

✶Prove that the quadrilateral is a parallelogram with PERPENDICULAR DIAGONALS.

Ex. 1 Quadrilateral *ABCD* has vertices *A*(2, 4), *B*(6, -3), *C*(-2, -2) and *D*(-6, 5). Using coordinate geometry, prove that *ABCD* is a rhombus.

Step 1: Plot and label the quadrilateral on the grid.



Step 2: Calculate the MIDPOINT of each diagonal and prove it's a parallelogram.

$\overline{AC}$:

$\overline{BD}$:

Step 3: Calculate the SLOPE of each diagonal and make a conclusion about the slopes.

$\overline{AC}$:

$\overline{BD}$:

Step 4: Write a conclusion about the quadrilateral.

ABCD is a Rhombus because it's a parallelogram with _____ diagonals.

Ex. 2 . Quadrilateral *ABCD* has vertices *A*(2, 4), *B*(1, -1), *C*(-4, -2), and *D*(-3, 3). Prove using coordinate geometry that *ABCD* is a rhombus.

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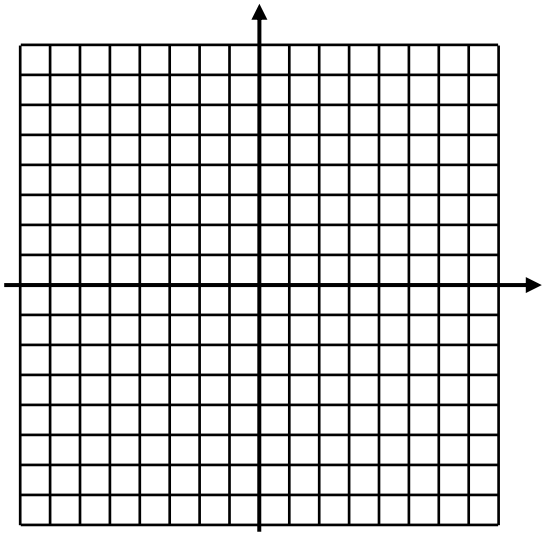
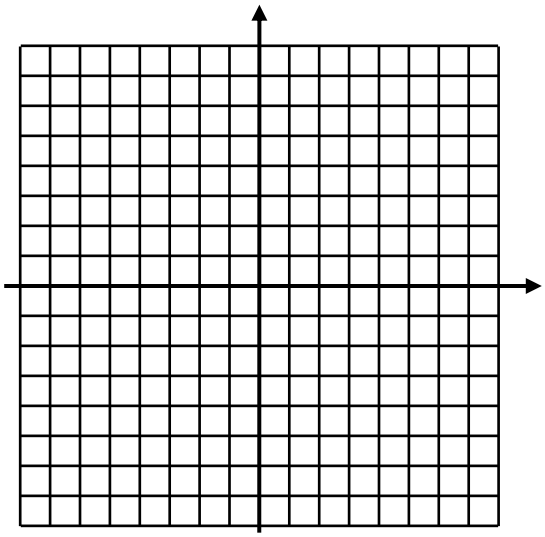
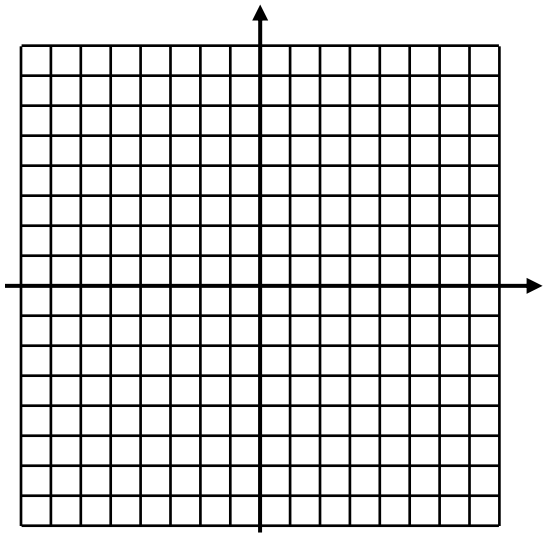


Ex. 3 Quadrilateral *PQRS* has vertices *P*(2, 4), *Q*(3, 1), *R*(-3, -1) and *S*(-4, 2). Prove using coordinate geometry that *PQRS* is **NOT** a rhombus.



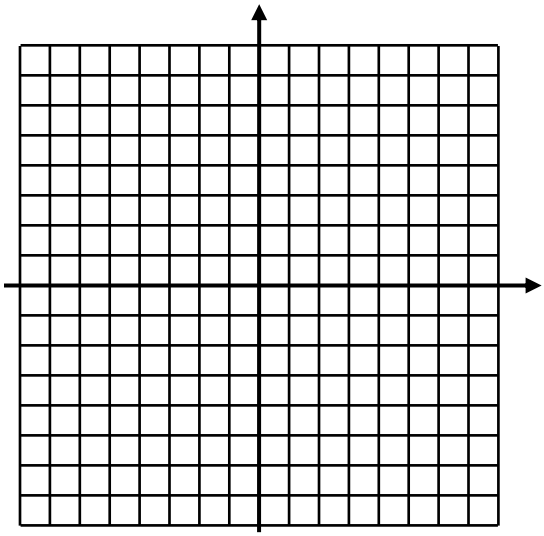
Proving Figures are Rhombii - Homework
Intro to Geometry

1. The vertices of quadrilateral BERT are B(-1,1), E(4,3), R(6,-2) and T(1,-4). Prove BERT is a rhombus.



2. Quadrilateral ABCD has vertices A(2, 4), B(1, 1), C(-3, -1) and D(-2, 2). Prove using coordinate geometry that:

(a) ABCD is a parallelogram.



(b) ABCD is *not* a rhombus.

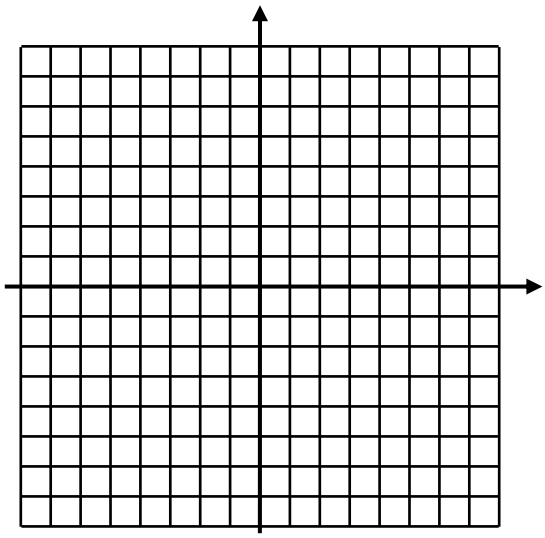
Proving Figures are Squares
Intro to Geometry

AIM: → To Prove a Quadrilateral is a Square using the diagonals of the quadrilateral.

To Prove that a Figure is a Square

✶Prove that the quadrilateral is a parallelogram with diagonals BOTH perpendicular AND congruent..

Ex. 1 Show that quadrilateral PARC is a square if it has coordinates $P(-3,0), A(0,3), R(-3,6), C(-6,3)$



Step 1: Plot and label the quadrilateral on the grid.

Step 2: Calculate the MIDPOINT of each diagonal and prove it's a parallelogram.

$\overline{PR}$:

$\overline{AC}$:

Step 3: Calculate the SLOPE of each diagonal and prove PARC is a rhombus.

$\overline{PR}$:

$\overline{AC}$:

Step 4: Calculate the DISTANCE of each diagonal and prove PARC is a rectangle.

$\overline{PR}$:

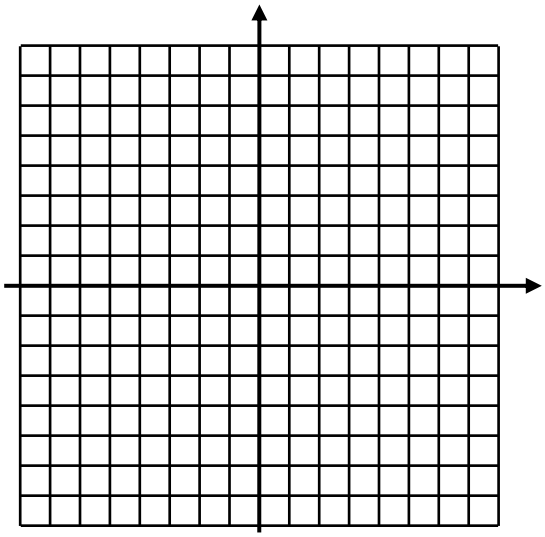
$\overline{AC}$:

Step 4: Make a conclusion about PARC:

Since PARC has the properties of a parallelogram, rhombus and a rectangle, it must be a

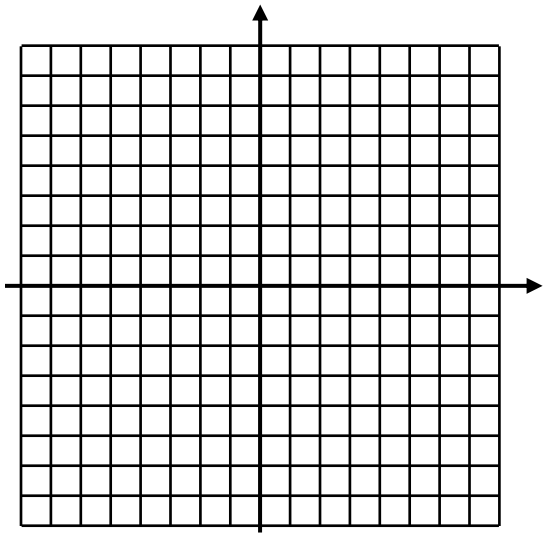
_____.

Ex. 2 . Quadrilateral KREJ has vertices K(1, -2), R(5, 0), E(3, 4) and J(-1, 2). Prove using coordinate geometry that KREJ is a square.



Proving Figures are Squares - Homework
Intro to Geometry

Quadrilateral RSTU has vertices R(5, 2), S(3, -2), T(-1, 0) and U(1, 4). Prove using coordinate geometry that RSTU is a square.



a.) Show it is a **parallelogram** by using MIDPOINT

b.) Show it is a **rhombus** by using SLOPE

c.) Show it is a **rectangle** by using DISTANCE

d.) Write a conclusion.

Properties of Parallelograms and Rectangles Practice
Intro to Geometry

AIM: → To define quadrilateral
→ To define parallelogram and state its special properties

Parallelogram : A quadrilateral with ***two*** pairs of ***parallel*** sides.

Recall

Important Properties that Follow from the Parallel Lines of a Parallelogram

1. Opposite angles of a parallelogram are congruent (equal in measure).

2. Consecutive angles of a parallelogram are supplementary (sum to 180^o).

3. Opposite sides of a parallelogram are congruent (equal in measure).

4. Diagonals of a parallelogram bisect each other (have the same midpoint).

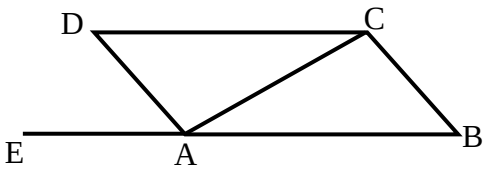
Solve each of the following problems. First, draw a diagram and properly label it.

1. In parallelogram PQRS, $PQ = 7x - 6$ and $RS = 5x + 12$, find x.

2. In parallelogram CDEF, side $\overline{\text{DE}}$ is extended through E to point G. If $m\angle FEG = 130°$, then find $\text{m}\angle\text{C}$.

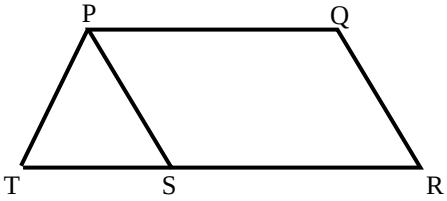
3. In parallelogram PQRS, diagonals $\overline{\text{PR}}$ and $\overline{\text{QS}}$ intersect at point M. If $QM = 6x + 18$ and $MS = 5x + 23$, then find QS.

4. $m\angle EAD = 45^\circ$ and $m\angle ACD = 40^\circ$. Find $m\angle CAD$.

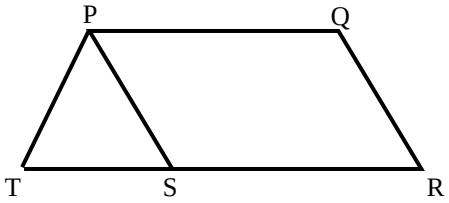


In the diagram at the right, PQRS is a parallelogram. Answer the following questions.

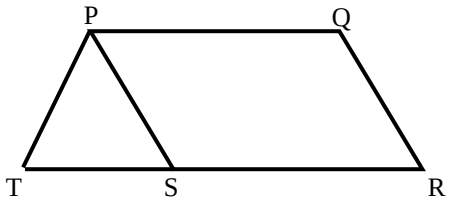
5. If $\overline{PT} \cong \overline{PS}$ and $m\angle Q = 125^\circ$, find $m\angle T$.



6. If $\overline{PT} \cong \overline{PS}$ and $m\angle TPS = 48^\circ$, find $m\angle Q$.



7. If $\overline{PT} \cong \overline{PS}$ and $m\angle QPS = 70^\circ$, find $m\angle TPS$.



Rectangles

Definition: A rectangle is a ***parallelogram*** with four right angles. It has all the properties of a parallelogram AND:

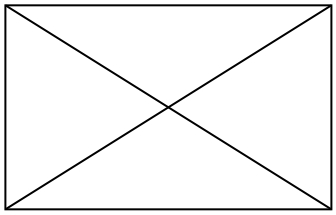
Properties of a Rectangle (To memorize!):

1.

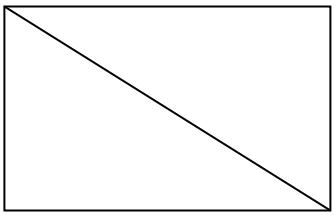
2.

As a reminder, a rectangle has all of the properties of a parallelogram (on page 18) *in addition to* the ones above.

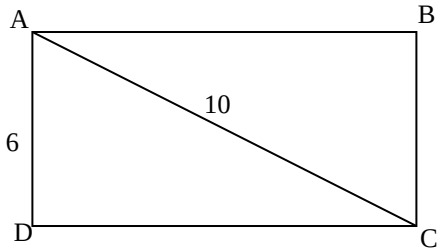
1. A rectangle has side lengths of 6 and 8. Find the length of one of its diagonals.



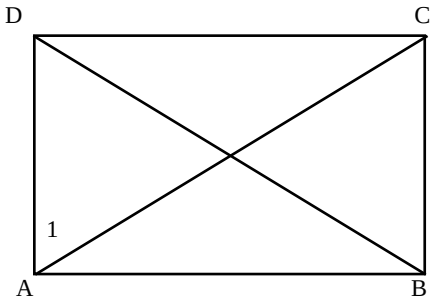
2. A rectangle has a diagonal of length 26 and a side of length 10. Find the other side. Then find the perimeter.



3. Rectangle *ABCD* is pictured below. Find DC and the perimeter of the rectangle..

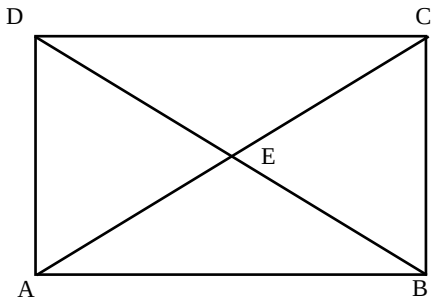


4. In rectangle ABCD, $m\angle 1 = 55^\circ$, find the $m\angle ABD$.



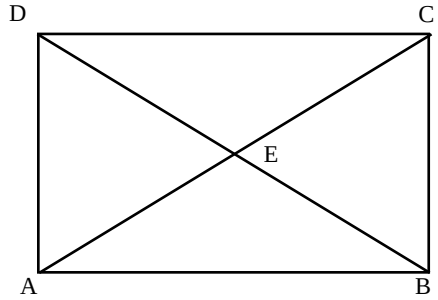
5. If AC = 5x – 2 and BD = 7x – 14, find x.

6. If AD is represented by 3x + 2 and BC is represented by 5x – 6, find x AND the length of AD.



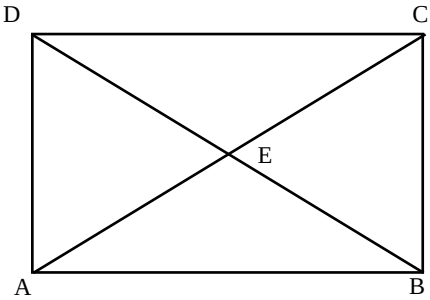
x=_____ AD=_____

7. If AC = 7x + 4 and BD = 5x + 10, find x AND the length of AC.

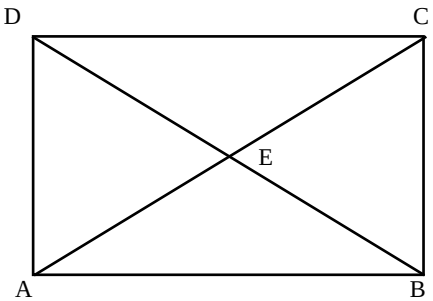


x=_____ AC=_____

8. If $m\angle DAC = 2x + 6$ and $m\angle CAB = 3x + 24$, find x, $m\angle DAC \,\&\, m\angle CAB$.

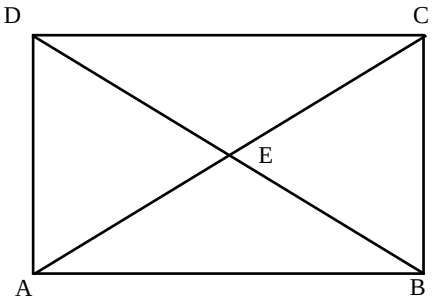


9. If CB:AB is 3:7 and the perimeter of the rectangle is 80, find CB and AB.

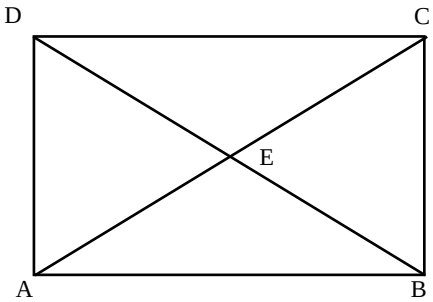


Quadrilaterals & Properties of Parallelograms and Rectangles – Homework
Intro to Geometry

1. If AB is 4x – 2, CB is 3x + 1 and the perimeter of the rectangle is 68, find x.

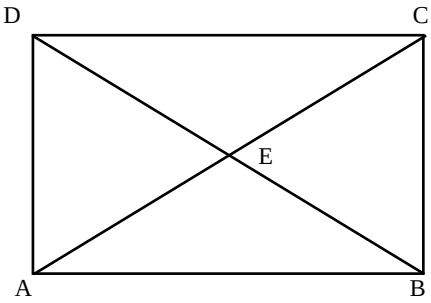


2. If If $m\angle DBA = 3x + 12$ and $m\angle DBC = 5x - 2$, find x, $m\angle DBA$ & $m\angle DBC$.



x= $m\angle DBA =$ $m\angle DBC =$

3a. If AB = 24 and AC = 25, find the length of BC.



3b. Use the information from 3a to find the perimeter of rectangle ABCD.

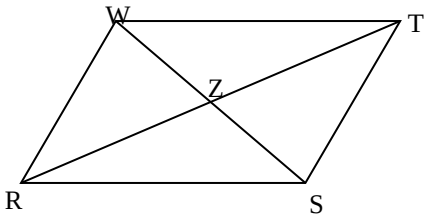
4. In parallelogram ABCD, $m\angle A = 2x + 40$ and $m\angle C = 6x - 10$. Find the number of degrees in $\angle A$ and $\angle B$.

$\angle A =$ _____

$\angle B =$ _____

5. In parallelogram CDEF, side $\overline{DE}$ is extended through E to point G. If $m\angle FEG = 156°$, then find $m\angle C$.

Given that WRST is a parallelogram, find the value of x or y in each question.



6. RS = 2x + 7, TW = 25, WR = 16

7. WZ = 4x – 3, ZS = 13

8. RZ = 17, ZT = 7y + 3

9. $m\angle$ WRS = 24x, $m\angle$ STW = 15x + 27

10. $m\angle$ WRS = 75, $m\angle$RST = 7x

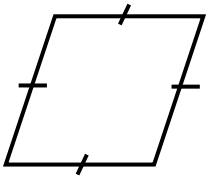
Quadrilaterals & Properties of a Rhombus
Intro to Geometry

***AIM*: → To define a rhombus and state its special properties**

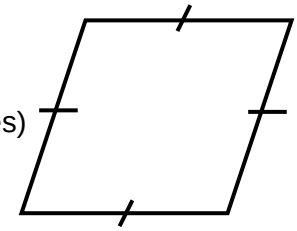
Definition: A rhombus is a ***parallelogram*** with four congruent sides (equilateral). It has all the properties of a parallelogram AND:

Properties of a Rhombus (To memorize!):

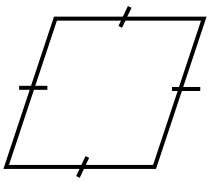
1. Equilateral (all 4 sides congruent)



2. The diagonals of a rhombus are perpendicular to each other (forming right triangles)



3. The diagonals of a rhombus each bisect a pair of opposite angles.



As a reminder, a rectangle has all of the properties of a parallelogram (below) *in addition to* the ones above.

Important Properties that Follow from the Parallel Lines of a Parallelogram

1. Opposite angles of a parallelogram are congruent (equal in measure).

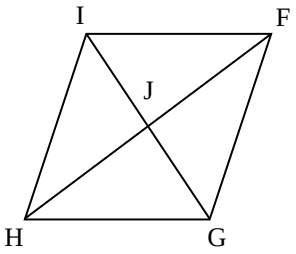
2. Consecutive angles of a parallelogram are supplementary (sum to 180^{o}).

3. Opposite sides of a parallelogram are congruent (equal in measure).

4. Diagonals of a parallelogram bisect each other (have the same midpoint).

On the next page we'll solve problems based upon these properties.

1. Use the diagram to the right to answer the following questions:



a) IF = 15.3, FG = _____

b) GH = 13, IF = _____

c) IH = 12, perimeter of the rhombus = _____

d) If FJ =6, JH = _____

e) If IJ = 3, IG = _____

f) $m\angle IJF =$_____

g) If $m\angle IFG = 40$, find $m\angle IFJ$ _____

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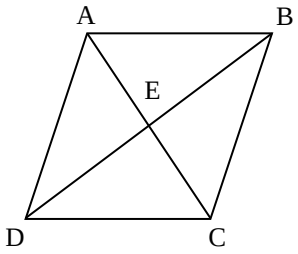
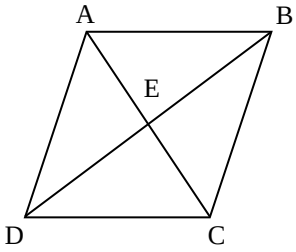
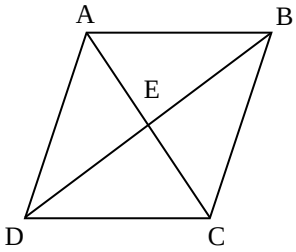
2. If AB = 4x + 1 and BC = x + 16, find x, AB and BC.



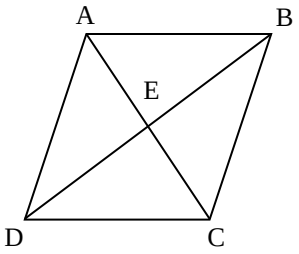
3. If AD = 2x – 4 and AB= x + 6, find x and the perimeter of the rhombus.



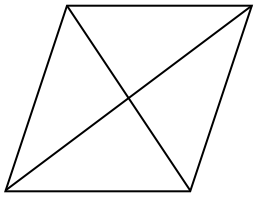
4. BE = 3x – 6 and BD = 8x – 28, find x, BE and BD.



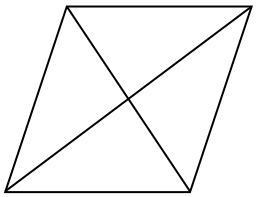
5. If $m\angle ABE = 2x + 8$ and $\angle CBE = 4x - 20$, find x, $m\angle ABE$ and $m\angle ABC$.



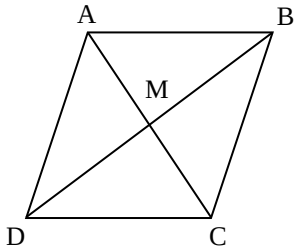
6. A rhombus has diagonals of length 10 and 24. What is the length of one of its sides?



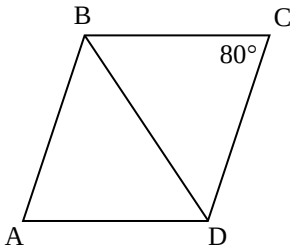
7. A rhombus has diagonals of length 4 and 8. Find its perimeter to the nearest tenth.



8. In rhombus ABCD, the diagonals intersect at point M. If $m\angle ADC = 70^\circ$, find $m\angle MCD$.

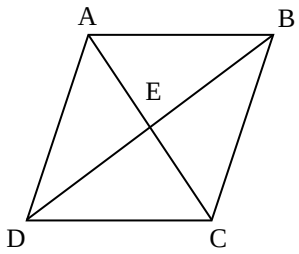


9. In the diagram of rhombus ABCD, $m\angle BCD = 80^\circ$. Find $m\angle BDA$.

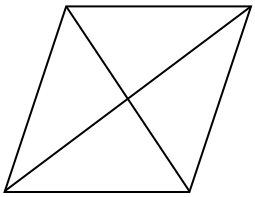


Quadrilaterals & Properties of a Rhombus – Homework
Intro to Geometry

1. If AB = 12x – 15 and BC = 10x – 11 find x, AB and BC.

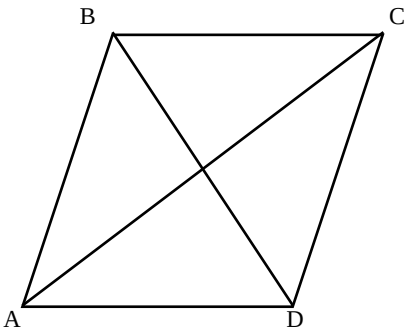


2. A rhombus has diagonals of 6 and 8; find the length of a side and then the perimeter of the rhombus.



3. In the following diagram, *ABCD* is a rhombus. If $m\angle BAD = 64^\circ$ then:

a. Find $m\angle BAC$



b. Find $m\angle ADC$

c. Find $m\angle BDA$

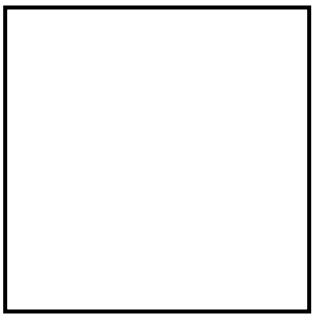
Quadrilaterals & Properties of Squares
Intro to Geometry

AIM: → To define a square and state its special properties

Square

A square is a ***parallelogram*** with four congruent sides (equilateral) and four congruent angles (equiangular). It has all the properties of a parallelogram, rectangle AND a rhombus. (Yes- just about everything goes!)

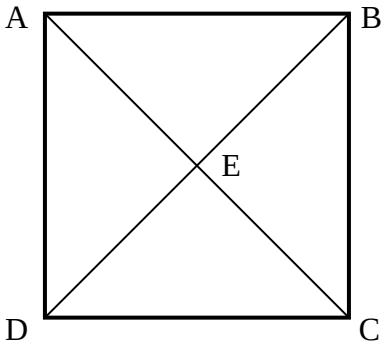
Properties of a Square:



1. All interior angles are right angles.
2. Diagonals are congruent (equal length).
3. All sides are congruent (equal length).
4. Diagonals are perpendicular to one another.
5. Diagonals bisect interior angles. (All bisected interior angles congruent)

Practice Problems Using the Properties of a Square

1. Use the square to the right to answer the following questions:



a) $m\angle ABC, m\angle BCD, m\angle CDA, m\angle DAB$ $m\angle AEB, m\angle BEC, m\angle CED, m\angle DEA$ are all: _____

b) $m\angle EAD, m\angle EAB, m\angle EBA, m\angle EBC$ $m\angle ECB, m\angle ECD, m\angle EDC, m\angle EDA$ are all: _____

c) If AB = 12, BC = _____

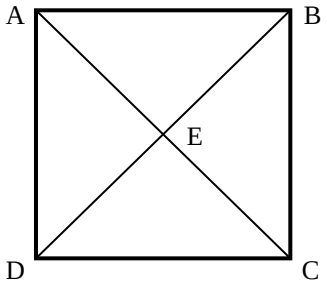
d) If BD = 13.2, AC = _____

e) If DE = 6, EB = _____

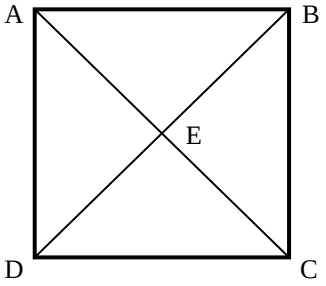
f) If DE = 6, EC =_____

g) If AC = 16, EB = _____

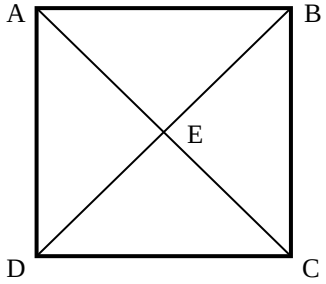
2. If AE = 5x +2 and DE = 7x – 4, find the value of x.



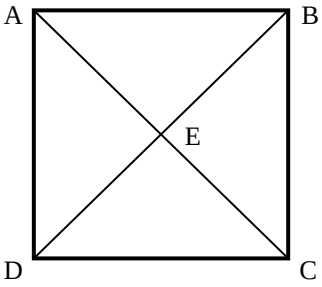
3. If CB = 6x + 1 and AB = 8x – 7, find the value of x AND the perimeter of the square.



4. If DE = 7x – 6 and DB = 16, find the value of x.



5. If $m\angle ECD = 3x + 12$, find the value of x.



6. Find in *simplest radical* form the length of the diagonal of a square whose perimeter is 20.

7. A square has a diagonal of length $3\sqrt{2}$. Find the perimeter of the square

Quadrilaterals & Properties of a Square – Homework
Intro to Geometry

Answer each question as True or False (review):

_____ 1. A rhombus is a parallelogram with four congruent sides.

_____ 2. A rectangle is a parallelogram with four right angles.

_____ 3. A rhombus is always a square.

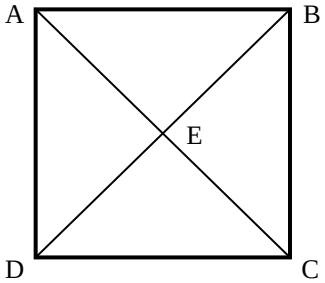
_____ 4. Every parallelogram has congruent diagonals.

_____ 5. In a rectangle, the diagonals are perpendicular.

_____ 6. If a quadrilateral is a square, it is also a rectangle.

Use the diagram to the right to answer the following questions:

7. Which segments are congruent to segment AB?



8. Which segments are congruent to segment AE?

9. Which angles are congruent to $\measuredangle ADE$?

10. If AD = 4, find the perimeter of the square.

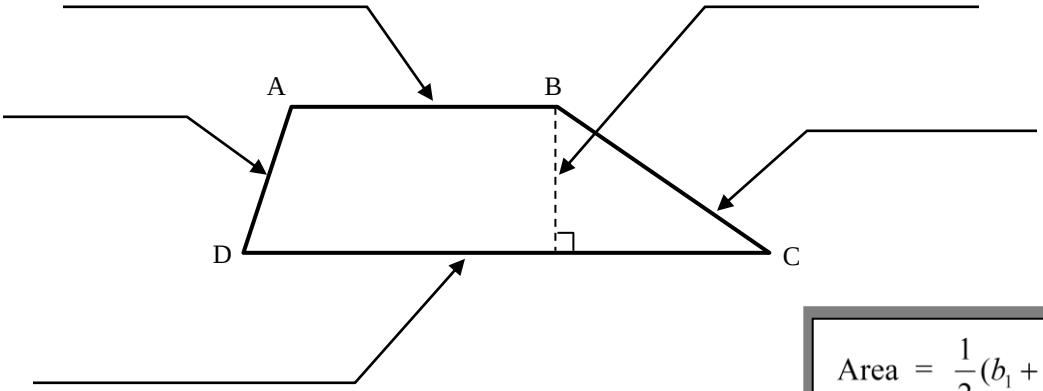
11. If AD = 4, find the length of a diagonal in simplest radical form (look at question 6 from the notes).

12. If AD = 4, find the length of AE, EC, ED and EB (HINT: you will need your answer from #11).

Properties of Trapezoids
Intro to Geometry

AIM: → *To define trapezoid and isosceles trapezoid*
→ *To state the properties of these quadrilaterals and use them to solve problems*

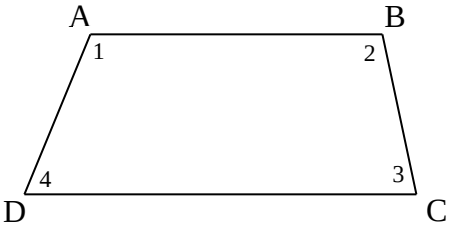
Trapezoid : A quadrilateral with only **one** pair of parallel sides.



$$\text{Area} = \frac{1}{2}(b_1 + b_2) \cdot h$$

✶ How do trapezoids and parallelograms differ?

Because a trapezoid has only 1 pair of opposite sides parallel, there are some differences in angle properties from the other quadrilaterals we've looked at:



Opposite angles are NOT congruent.

Only 2 pairs of consecutive angles are supplementary.
(1 & 4 are supplementary as are 2 & 3).

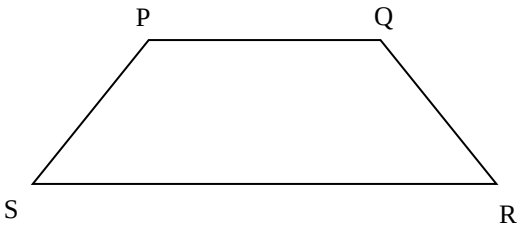
Name the two bases of trapezoid ABCD:_____.

Name the two legs of ABCD:_____.

If $m\angle 1 = 105^\circ$, find $m\angle 4$:_____.

Isosceles Trapezoid : A trapezoid whose legs are congruent (same length).

$\overline{PS} \cong \overline{QR}$



Important Properties of an Isosceles Trapezoid

1. Legs are congruent (have the same length).

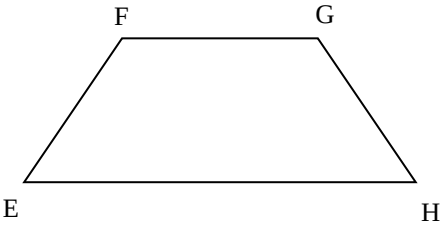
2. Base angles are congruent (have the same measure).

3. Diagonals are congruent (have the same length).

Isosceles Trapezoid EFGH has legs EF and GH:

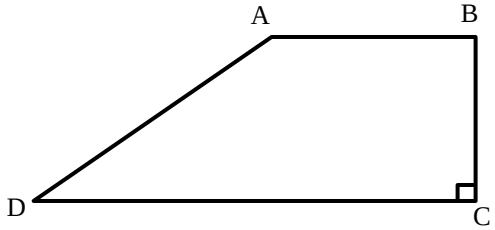
If EF = 5, GH = _____

If $m\angle E = 45$, find $m\angle F$ & $m\angle G$:_____



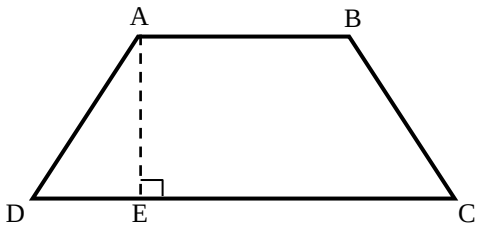
Some of the practice problems will become more difficult as we add in diagonals.

1. In the diagram below, *ABCD* is a trapezoid with *AB* = 7, *DC* = 10, and *BC* = 2.



Find AD in ***to the nearest hundredth***.

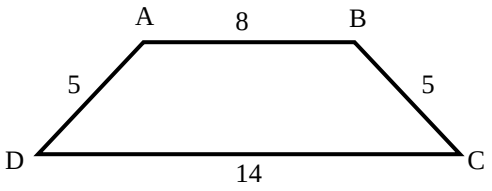
2. In the diagram below, *ABCD* is an isosceles trapezoid with *AD* = *BC* = 10, *AB* = 7, and *DC* = 23. Altitude *AE* is drawn.



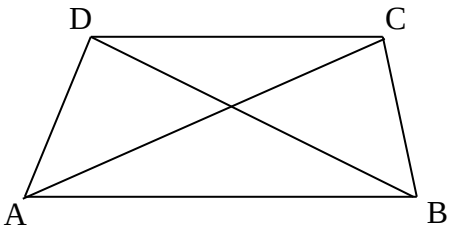
Find *AE*.

3. In the diagram below, *ABCD* is an isosceles trapezoid. *AB* = 8, *DC* = 14, and *AD* = *BC* = 5. Find the area of *ABCD.*

Area of a trapezoid: $\frac{(b_1 + b_2)h}{2}$



4. Given trapezoid ABCD with diagonals AC and BD and legs AD and BC, find the following:



a) If $m\angle DCA = 30^\circ$, what is $m\angle CAB$?

b) If $m\angle ADC = 110^\circ$, what is $m\angle DAB$?

c) If $m\angle ABC = 27^\circ$, what is $m\angle DCB$?

Properties of Trapezoids – Homework
Intro to Geometry

1. Answer True or False for each of the following statements:

a) A trapezoid has two bases. _____

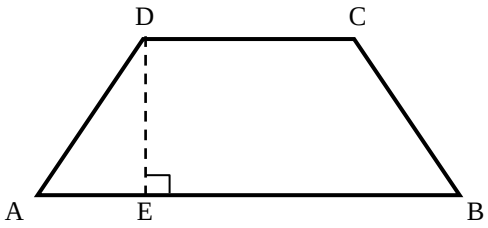
b) A trapezoid may have a right angle. _____

c) A trapezoid may have 3 congruent sides. _____

d) The bases of an isosceles trapezoid are congruent. _____

e) The parallel sides of a trapezoid are called the legs. _____

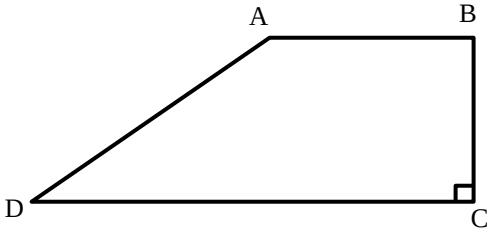
2. In the diagram below, *ABCD* is an isosceles trapezoid with altitude $\overline{DE}$ drawn. DC = 7, AB = 13, and DE = 4.



(a) Find the length of AD.

(b) Find the perimeter of trapezoid ABCD.

3. In the diagram below *ABCD* is a trapezoid with *AB* = 5, *DC* = 13 and *DA* = 17.



Find the length of BC.

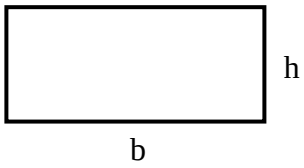
Finding Areas of Polygons Using Coordinate Geometry
Intro to Geometry

AIM: → To review area formulas of geometric figures
→ To calculate the areas of closed figures on coordinate axes.

Do Now

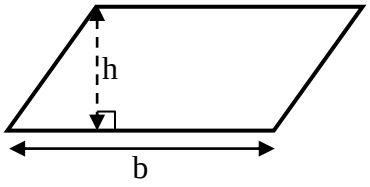
For each figure below, write down the formula for finding its area.

Rectangle



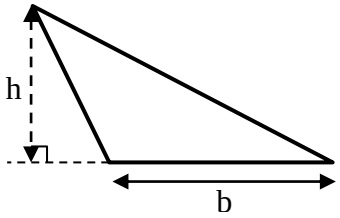
Area =

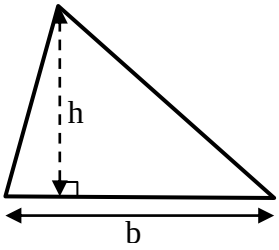
Parallelogram



Area =

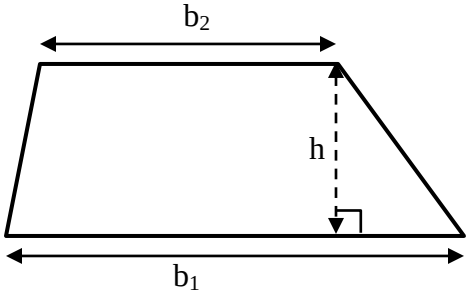
Triangle

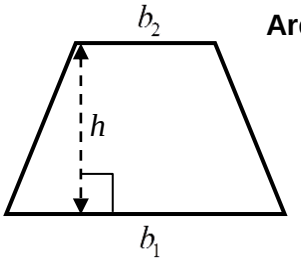




Area =

Trapezoid





Area =

Notice the similarity of the formulas. They all involve multiplying the base(s) and the height.

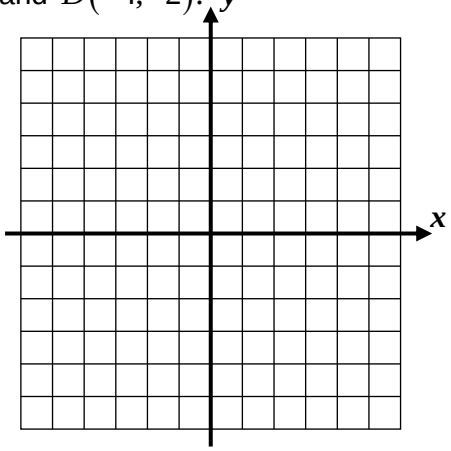
When finding the areas of polygons in the coordinate plane, we need to consider **two** scenarios:

Scenario (1) *The sides of the polygon are parallel to the axes.*

If the polygon is graphed such that its sides (or segments within) are drawn on the grid lines of the graph paper, then we can count the lengths and apply our area formulas.

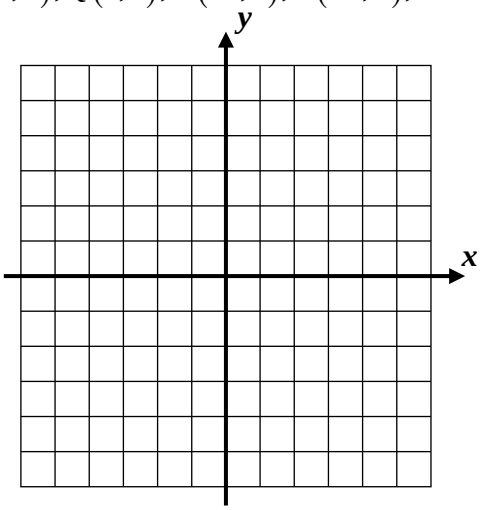
Ex. 1 Trapezoid *ABCD* has vertices $A(1,4), B(4,4), C(5,-2),$ and $D(-4,-2)$.

Find the area of *ABCD*.

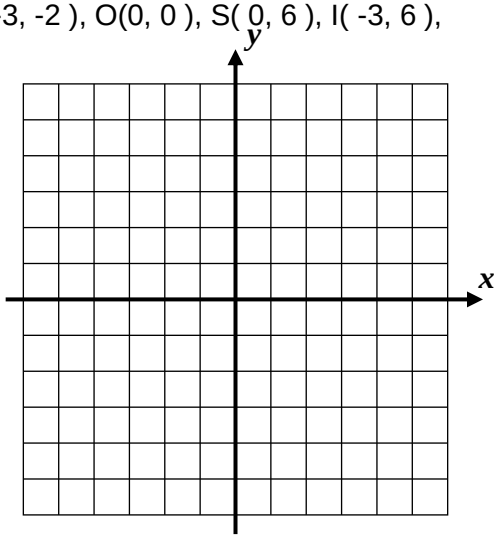


Some figures may be subdivided into smaller regions so that common area formulas can be used to calculate the overall area.

Ex. 2 Find the area of polygon *PQRSTU* with coordinates $P(4,0), Q(3,3), R(-1,3), S(-4,0),$ $T(-1,-3),$ and $U(3,-3)$.



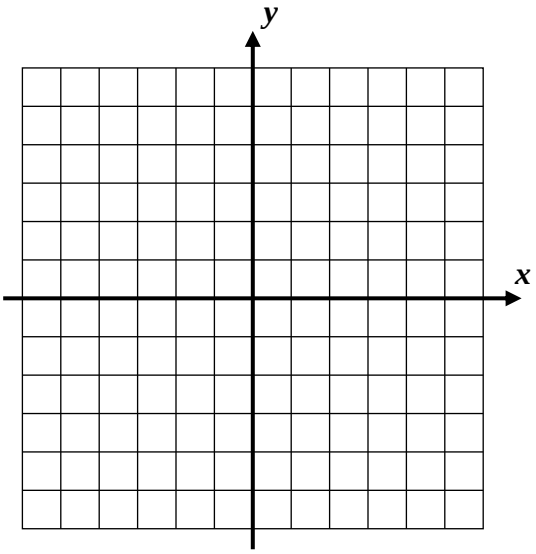
Ex. 3 Find the area of polygon *COSINE* with coordinates C(-3, -2), O(0, 0), S(0, 6), I(-3, 6), N(-6, 4)and E(-6, 0).



Scenario (2) *The sides of the polygon are <u>not</u> parallel to the axes.*

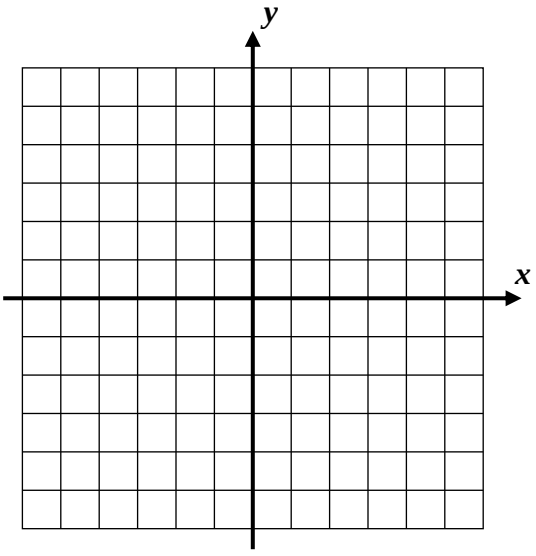
If the polygon is graphed such that its sides (or segments within) are *not* drawn on the grid lines of the graph paper, then we can enclose the figure in a "box" to determine its area.

<u>Ex. 4</u> Find the area of triangle *XYZ* if it has vertices $X(2,4), Y(6,3),$ and $Z(-2,-2)$.



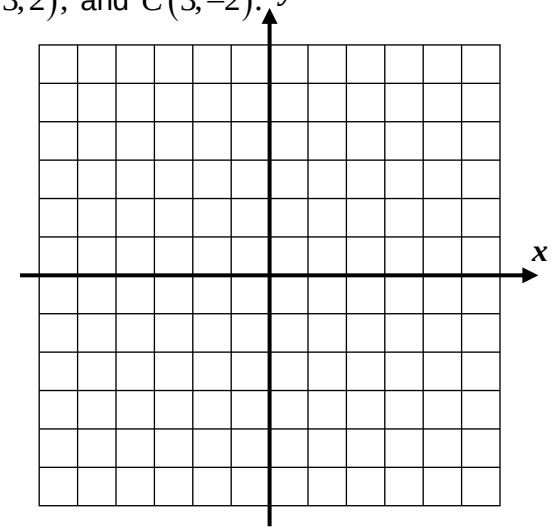
<u>Hint:</u> Surround the figure with a "box" and subtract the areas of the parts outside of the graphed polygon

<u>Ex. 5</u> Find the area of polygon SMILE if it has vertices $S(5,1), M(3,5), I(-4,5), L(-5,-2)$ and $E(-1,-6)$.



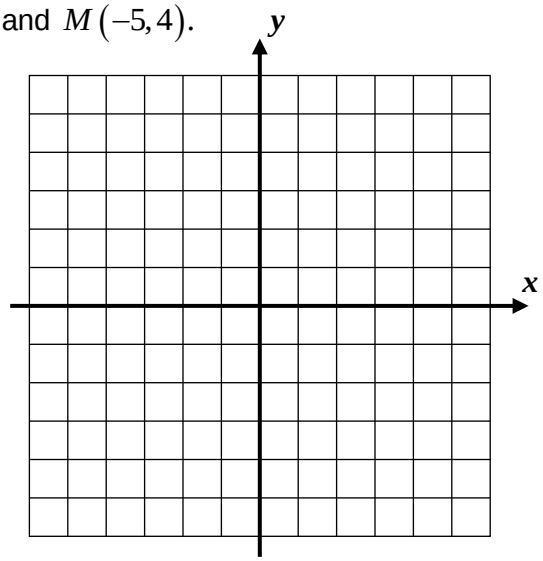
Class Practice & Homework

1. Find the area of triangle *ABC* if it has vertices $A(-4,2), B(3,2),$ and $C(3,-2).$

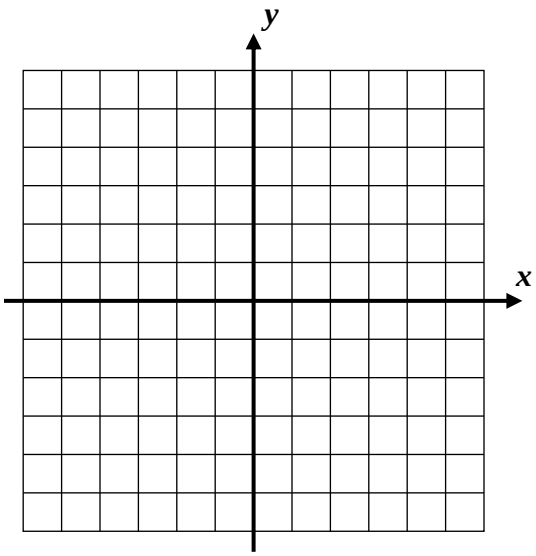


2. Trapezoid *JKLM* has vertices $J(5,4), K(3,-2), L(-1,-2),$ and $M(-5,4).$

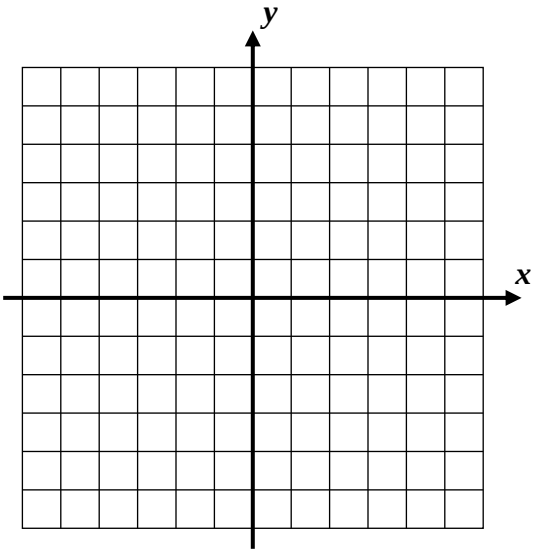
(a) Find the area of *JKLM*.



3. Triangle *ACE* has vertices $A(3,-2), C(1,3),$ and $E(-3,1)$. Find the area of *ACE*.



4. Find the area of trapezoid *PLUS* if it has vertices $P(-2,4), L(4,-2), U(-1,-4),$ and $S(-4,-1)$.



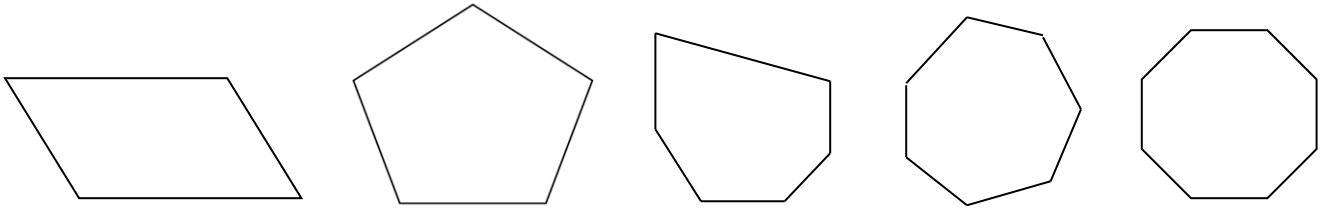
Interior & Exterior Angles of Polygons
Intro to Geometry

AIM: → To find the sum of the measures of the interior angles of a polygon
→ To find the sum of the measures of the exterior angles of a polygon.

*We will be studying convex polygons only.

This scallop shell resembles a 13-sided polygon with diagonals drawn from one of the vertices. A diagonal connects any two nonconsecutive vertices.

Polygons with more than three sides have diagonals. Show all possible diagonals drawn from one vertex:



quadrilateral pentagon hexagon heptagon octagon

What shapes form as you separate the polygons with diagonals? How does this help us find the measures of the angles inside of the polygons?

Complete the table below:

Convex Polygon	# of Sides	# of Triangles	**Sum of Angle Measures**
Triangle			
Quadrilateral			
Pentagon			
Hexagon			
Heptagon			
Octagon			

Look for a pattern in the sum of the angle measures. How could we use it to find the sum of the interior angles for a polygon with *n* sides (*n*-gon)?

Interior Angle Sum Theorem
If a convex polygon has *n* sides and *S* is the sum of the measures of its interior angles, then

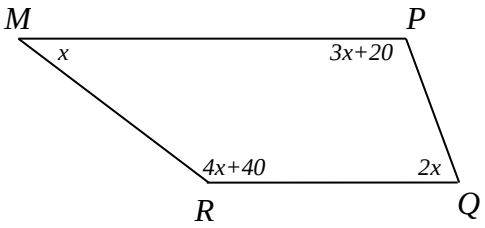
Practice:

1. Find the sum of the measures of the interior angles of each convex polygon.

(a) 32-gon (b) 19-gon (c) 40-gon

2. Find the measure of x using the given information.

(a)

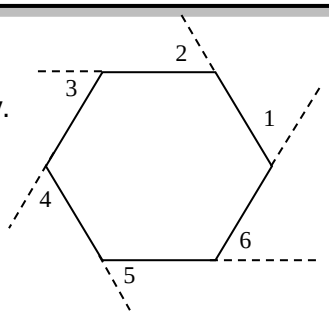


(b) decagon in which the measures of the interior angles are $x+5, x+10, x+20, x+30, x+35, x+40, x+60, x+70, x+80,$ and $x+90$.

3. How many sides does a polygon have if the sum of its angle measures is $2700°$?

Exterior Angle Sum Theorem
For any convex polygon, the sum of the measures of the exterior angles, one at each vertex, is $360°$.

At any vertex of a polygon, an exterior angle forms a linear pair with the interior angle. The interior angle and the exterior angle are **supplementary**.



Practice:

1. What is the sum of the measures of the exterior angles of a decagon?

2. Find the measure of an exterior angle of convex regular octagon *ABCDEFGH.*

3. Find the measure of one exterior angle of a polygon that has:

   (a) 4 sides (b) 10 sides (c) 36 sides

4. The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

   (a) 60 (b) 90

5. In quadrilateral *ABCD*, $m\angle A = x, m\angle B = 2x - 12, m\angle C = x + 22,$ and $m\angle D = 3x.$
   (a) Find the measure of each interior angle of the quadrilateral.

   (b) Find the measure of each exterior angle of the quadrilateral.

6. The measure of an exterior angle of a regular polygon is $45°$.
   *Note: A regular polygon is both equilateral and equiangular.

   (a) Find the number of sides of the polygon.

   (b) Find the measure of each interior angle.

   (c) Find the sum of the measures of the interior angles.

Interior & Exterior Angles of Polygons - HOMEWORK
Intro to Geometry

Directions: Answer each of the following questions. Make sure to read the questions carefully to determine what is being asked for.

1. What is the sum of the interior angles of a twelve-sided figure?

2. What is the sum of the exterior angles of a twelve-sided figure?

3. What is the measure of an interior angle of a regular octagon?

4. What is the sum of the interior angles of a decagon (10 sided figure)?

5. What is the measure of an interior angle of a regular 15-sided figure?

6. What is the sum of the exterior angles of a regular 20-sided figure?

7. What is the measure of an exterior angle of a regular 15-sided figure?

8. What is the measure of an interior angle of a regular 20-sided figure?

9. What is the sum of the interior angles of a 14-sided figure?

10. The measure of an exterior angle of a regular polygon is 45°. How many sides does this polygon have?

11. The measure of an interior angle of a regular polygon is 120°. How many sides does this polygon have?

12. The measure of an exterior angle of a regular polygon is 20°. How many sides does this polygon have?

13. The measure of an interior angle of a regular polygon is 140°. How many sides does this polygon have?