Name: ______________________________ **UNIT 10 BOOK**

Introduction to Functions
Intro to Geometry

AIM: → *To define a relation*
→ *To determine when a relation is a function using lists, tables, and graphs*
→ *To define domain and range*
→ *To state the domain and range of various functions*

Functions describe the relationship between two variables. This is one of the most important concepts in mathematics. In order to develop the definition of a function, we must first examine relations.

Relation A relation is a set of ordered pairs. Any group of numbers is a relation, so long as these numbers come in pairs.

Some examples of relations: $\{(0,5),(-12,36),(33,11)\}$

Write your own example: $\{ \quad \}$

$\{(-3,6),(1,8),(-65,2),(27,81)\}$

$\{(\phi,\sigma),(\pi,\varepsilon),(\theta,\mu),(\beta,\tau)\}$

These numbers are categorized as elements of the *domain* or *range* of the relation.

Domain : the set of all input values (or *x*-values) of the ordered pairs in a relation.

Range : the set of all output values (or *y*-values) of the ordered pairs in a relation.

Ex. 1 Given the relation: $\{(2,7),(-35,4),(71,-9),(0,42),(-6,11)\}$

Domain: $2,-35,71,0,-6$ Range: $7,4,-9,42,11$

It is customary to list the values as a set in ascending order from least to greatest:

Domain: $\{-35,-6,0,2,71\}$ Range: $\{-9,4,7,11,42\}$

Ex. 2 Given the relation $\{(-2,3),(-1,2),(0,1),(1,0),(-2,-1)\}$, state the domain and range.

Domain: Range:

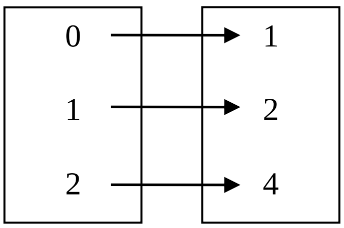
➪ *When is a relation a function?*

A ***function*** is a special relation, a rule, in which each member of the domain is paired with ***exactly*** one member of the range. It is often expressed using a table, graph, or equation.

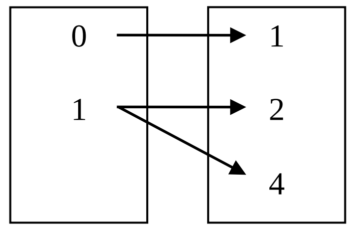
Using a table

Compare the two relations below:

Relation 1 : $\{(0,1),(1,2),(2,4)\}$



Relation 2 : $\{(0,1),(1,2),(1,4)\}$



Which relation is a function? How do you know?

Ex. 3 State whether or not the given relation represents a function. Also, state the domain and range of the given relation.

1. $\{(-2,3),(-1,2),(0,1),(1,0),(2,-1)\}$

(a) ____

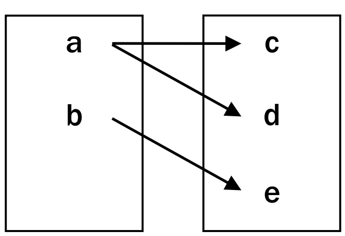
(b) D:____________________ R:____________________

2. $\{(0,0),(1,1),(2,2),(1,3),(0,4)\}$

(a) ____

(b) D:____________________ R:____________________

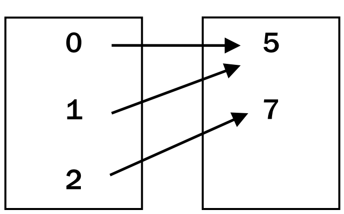
3.



(a) ____

(b) D:____________________ R:____________________

4.



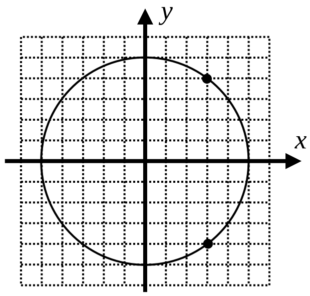
(a) ____

(b) D:____________________ R:____________________

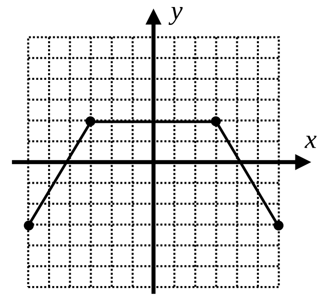
Using a Graph

The following two graphs represent relationships between the variables *x* and *y*. Answer the following questions based on the two graphs.

Graph #1



Graph #2



(a) What are the outputs of both relationships for the input *x* = 3?

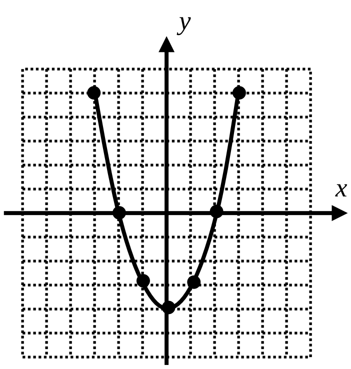
Graph 1　　　　Graph 2

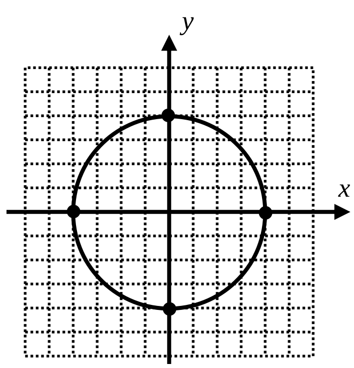
(b) In which of these graphs is *y* a function of *x*? Explain.

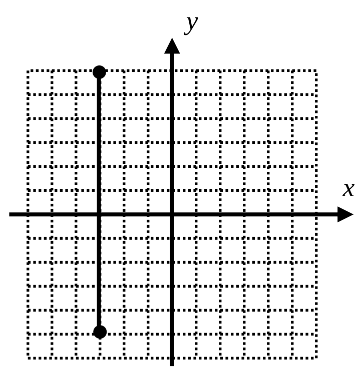
The Vertical Line Test – The Vertical Line Test is a quick way to see if the graph of a relationship is that of a function.

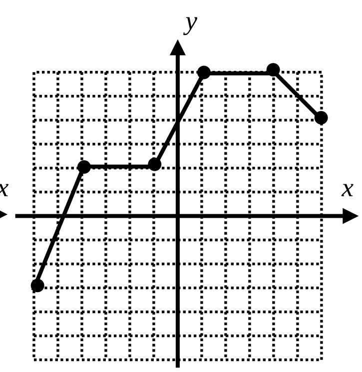
Vertical Line Test – If **any** vertical line intersects the graph of a relationship **more than once**, it is **NOT** the graph of a function because a given input has more than one output.

Ex. 4 Using this test, determine which two graphs below represent functions and which two do not.

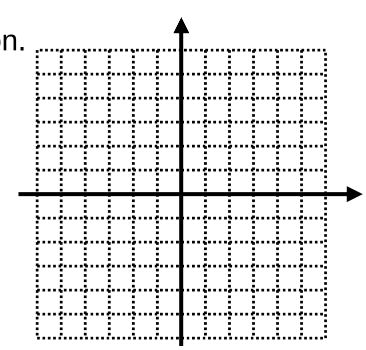








Ex. 5 Draw your own example and **show** that it *is* a graph of a function.



Using an Equation

The equation $y = 2x + 5$ gives the variable *y* as a function of the variable *x*. In this case the value of *y* depends on the value of *x*.

(a) What is the output of this function when *x* has the following input values?

(i) $x = 4$ (ii) $x = -6$ (iii) $x = 0$ (iv) $x = -3$

(b) For what value of *x* is the output of this function $y = 17$?

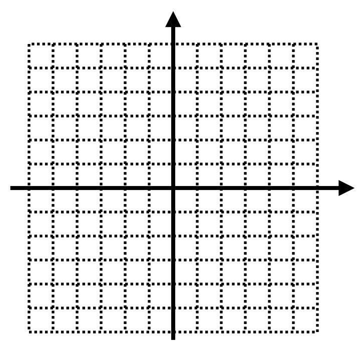
HOMEWORK

1. Answer the following questions below using the given relation: $\{(2,5),(3,10),(1,9),(5,9)\}$

   (a) What is the **domain** of the relation? (b) What is the **range** of the relation?

   (c) Is the relation a function? How do you know?

2. Sketch a graph of a relation that is also a function. **Show** how you know that the graph represents a function.



3. The following two tables show relationships between the variables *x* and *y*. One of them is a relationship in which *y* is a function of *x* and the other is not. Explain which represents the function and why.

Relation 1

x	*y*
0	-2
1	-1
2	2
3	7
4	14

Relation 2

x	*y*
0	2
1	14
0	4
1	19
0	6

Review of Solving Quadratics
Intro to Geometry

AIM: → *To recall factoring*
→ *To solve a quadratic equation*

Do Now

Solve the following:

a.) $5x-9=11$ b.) $8x+15=-9$ c.) $x^2-9=0$

What did you have to do to solve c?

:

RECALL: To factor a quadratic, you must come up with two numbers that *multiply* to the constant, and *add* to the middle value. Watch your signs!

Ex 1: Factor each of the following:

(a) $x^2-7x+10$ (b) $x^2+9x+18$ (c) $x^2+6x-16$

(d) $r^2+8r+16$ (e) $m^2+2m-24$ (f) $x^2+2x-15$

(g) w^2-w-42 (h) $x^2-10x+25$ (i) $y^2-18y+32$

TO SOLVE A QUADRATIC EQUATION   First, you must factor the quadratic equation. Then set each factor equal to zero and solve for each value.

Ex2: Solve each quadratic equation.

a.) $x^2 + x - 6 = 0$

b.) $x^2 - 7x - 18 = 0$

c.) $y^2 + 9y + 20 = 0$

d.) $x^2 - 7x + 12 = 0$

e.) $x^2 + 7x + 6 = 0$

f.) $x^2 + 12x + 36 = 0$

g.) $x^2 - 49 = 0$

h.) $x^2 - 25 = 0$

REVIEW : Solve each of the following inequalities:

a.) $2x - 5 > 7$

b.) $5x - 2 \le 13$

c.) $7 - 3x \ge 34$

Review of Solving Quadratics - HOMEWORK
Intro to Geometry

Solve each Quadratic Equation:

1. $x^2 - 11x + 30 = 0$

2. $x^2 - 7x - 30 = 0$

3. $x^2 + 9x - 36 = 0$

4. $x^2 + 13x + 40 = 0$

5. $x^2 - 49 = 0$

6. $x^2 - 36 = 0$

7. $x^2 + 10x + 25 = 0$

8. $x^2 + 4x - 60 = 0$

9. $x^2 - x - 2 = 0$

10. $x^2 + 4x - 12 = 0$

Finding the Domain of a Function Expressed as an Equation
Intro to Geometry

AIM: → **To recall the definition of domain**
→ **To find the domain of a function expressed as a formula**

Do Now

Find the domain of the following functions:

(a) $\{(-22,44),(2,6),(0,13)\}$

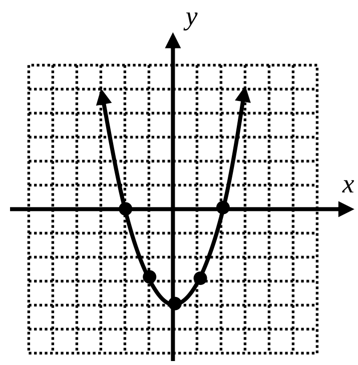
D:

(b)

x	y
2	7
5	10
6	11
9	14

D:

(c)



D:

If a function is expressed in list, table, or graph form, you can identify the domain by simply looking at the list, table, or graph. How do you find the domain of a function that is expressed as an equation?

You have learned two rules that will help us develop a strategy that will answer the above question:

1. You cannot divide by zero. (No zeros in the denominator of a fraction.)

2. You cannot take the square root of a negative number. (We are working with $\mathbb{R}$.)

- To find the domain of a function expressed as an equation, you need to find all input values that the function can take on without causing an undefined operation to occur.
- In other words, find all input values that *do not* cause you to divide by zero or take the square root of a negative number. The input values that cause these undefined operations are *not* in the domain.
- If there are no input values that cause undefined operations, then the domain is all real numbers ($\mathbb{R}$).

Ex. 1 Find the domain of each function specified below.

(a) $y=\dfrac{2x+1}{3}$

(b) $y=\dfrac{4}{x}$

(c) $y=\dfrac{2x}{x+6}$

(d) $y=\sqrt{2x-16}$

(e) $y=\dfrac{4}{x^2-36}$

(f) $y=x^2-3x-8$

(g) $y=|3x-9|$

(h) $y=\dfrac{2-x}{x^2-2x-8}$

(i) $y=\dfrac{5}{x^2-9}$

(j) $y=\sqrt{2-3x}$

(k) $y=\dfrac{x+1}{x^2-5x}$

(l) $y=\sqrt{x+5}$

(m) $y=4+\sqrt{5-2x}$

(n) $y=|x|+x$

(o) $y=\dfrac{x^2+x-2}{x^2-x-2}$

(p) $y=\sqrt{12x-66}+x$

(q) $y=x-22$

✷ What effect do you think the domain of a function has on its range?

Finding the Domain of a Function Expressed as an Equation
Intro to Geometry – Homework

Find the domain of each of the following equations.

1. $y = \dfrac{5x+3}{7}$

2. $y = \dfrac{5x}{x+1}$

3. $y = \dfrac{3}{x}$

4. $y = \sqrt{x-8}$

5. $y = 2x^2 - 5x - 10$

6. $y = \dfrac{8}{x^2 - 16}$

9. $y = \dfrac{x+3}{x^2 - 2x}$

10. $y = \sqrt{x+3}$

11. $y = |2x-8|$

12. $y = |x| + 11$

13. $y = \dfrac{5-x}{x^2 - 5x - 14}$

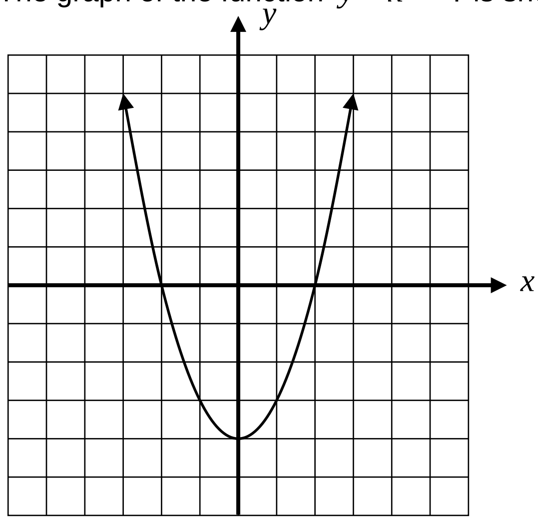
14. $y = 5x - 18$

Examining Graphs of Functions
Intro to Geometry

AIM: → ***To examine graphs of functions and identify domain, range, maximum & minimum values, and intervals where the graph is increasing or decreasing***
→ ***To sketch a graph given the above items as criteria***

Do Now

The graph of the function $y = x^2 - 4$ is shown below.



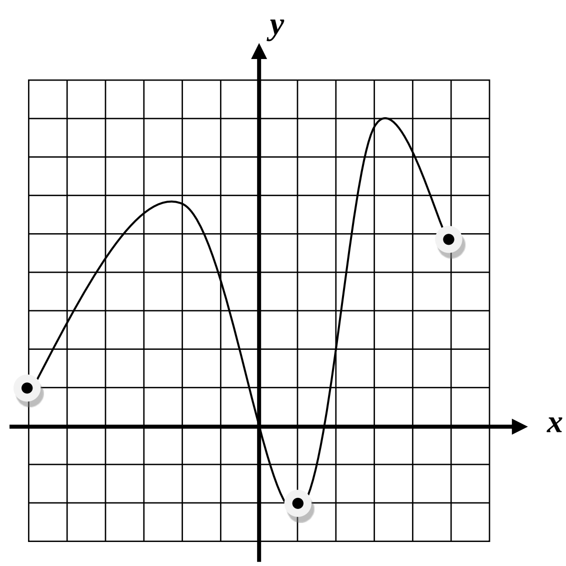
(a) From this graph, identify the domain and range of the function.

Domain:

Range:

(b) What is the turning point of the graph? Is it a maximum or a minimum?

Ex. 1



(a) What is(are) the *y*-intercept(s) of the graph of the function?

(b) What is(are) the *x*-intercept(s) of the graph of the function?

(c) What is the domain of the function?

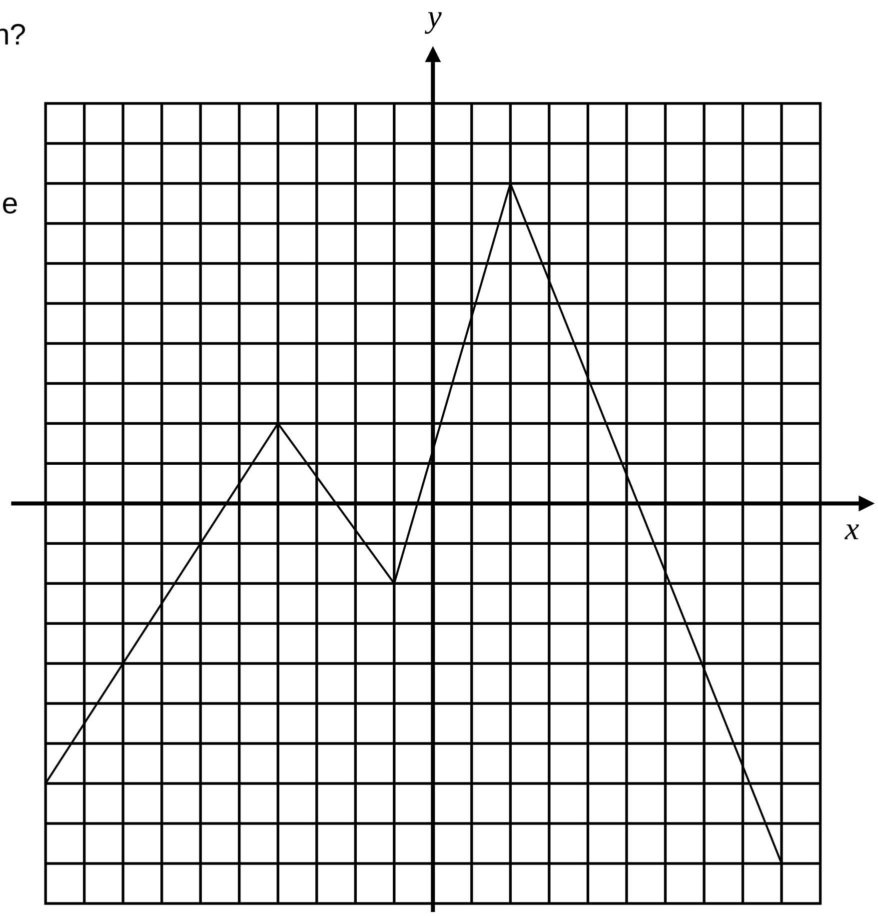
(d) What is the maximum value of the function?

(e) What is the minimum value of the function?

(f) What is the range of the function?

Ex. 2

(a) What is the domain of the function?



(b) How many turning points does the graph of the function have?

(c) What is the maximum value of the function?

(d) What is the *x*-coordinate of the point where the function is a maximum?

(e) What is the minimum value of the function?

(f) What is the *x*-coordinate of the point where the function is a minimum?

(g) What is the range of the function?

(h) On which interval below is the graph of the function <u>increasing</u>?

(1) $-4 < x < -1$ (3) $-10 < x < -4$

(2) $-1 < x < 5$ (4) $2 < x < 9$

<u>Ex. 3</u>

Using the graph grid below, sketch the graph of a function that satisfies the given criteria.

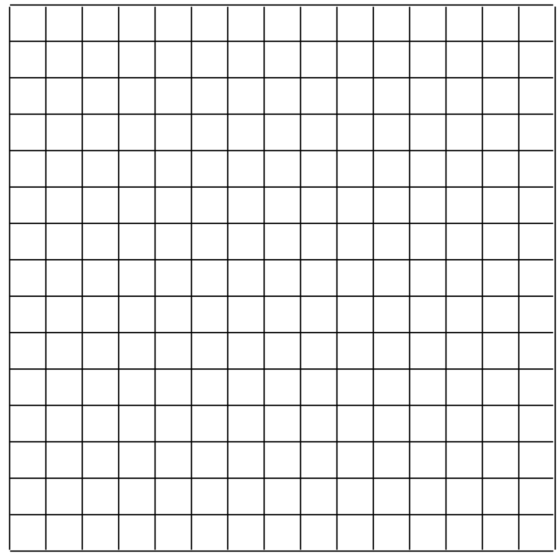
➡ *Don't forget to draw and label your axes!*

Domain: $-4 \le x \le 5$

Range: $-2 \le y \le 4$

Increasing: $-4 < x < -1$; $1 < x < 3$

Decreasing: $-1 < x < 1$; $3 < x < 5$



Review

Determine the domain of each of the functions from its formula below.

(a) $f(x) = \dfrac{x}{x-10}$

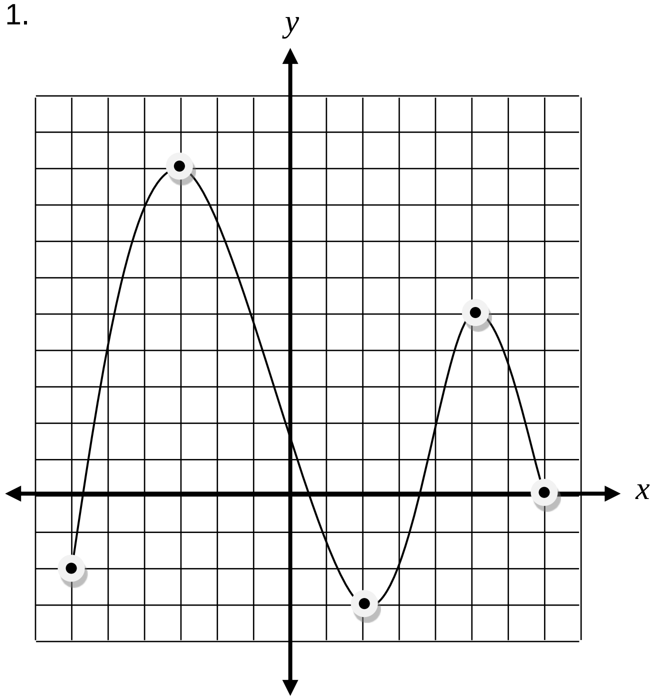
(b) $y = x^2 - 1$

(c) $k(x) = \sqrt{x+2} - 7$

(d) $h(x) = \dfrac{x+1}{x^2-16}$

Examining Graphs of Functions - HOMEWORK
Intro to Geometry

1.



(a) What is(are) the y-intercepts of the graph of the function?

(b) What is(are) the x-intercepts of the graph of the function?

(c) What is the maximum value of the function?

(d) What is the minimum value of the function?

(e) What is the range of the function?

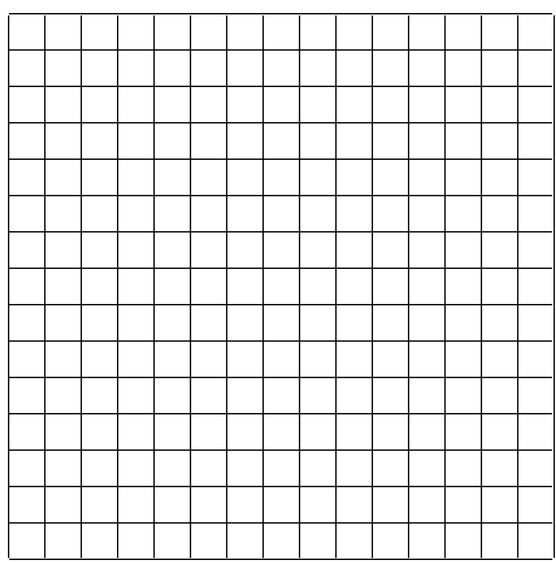
2. Using the graph grid below, sketch the graph of a function that satisfies the given criteria.
➡ *Don't forget to draw and label your axes!*

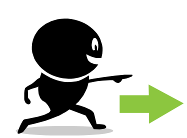
Domain: $-3 \le x \le 3$

Range: $-4 \le y \le 4$

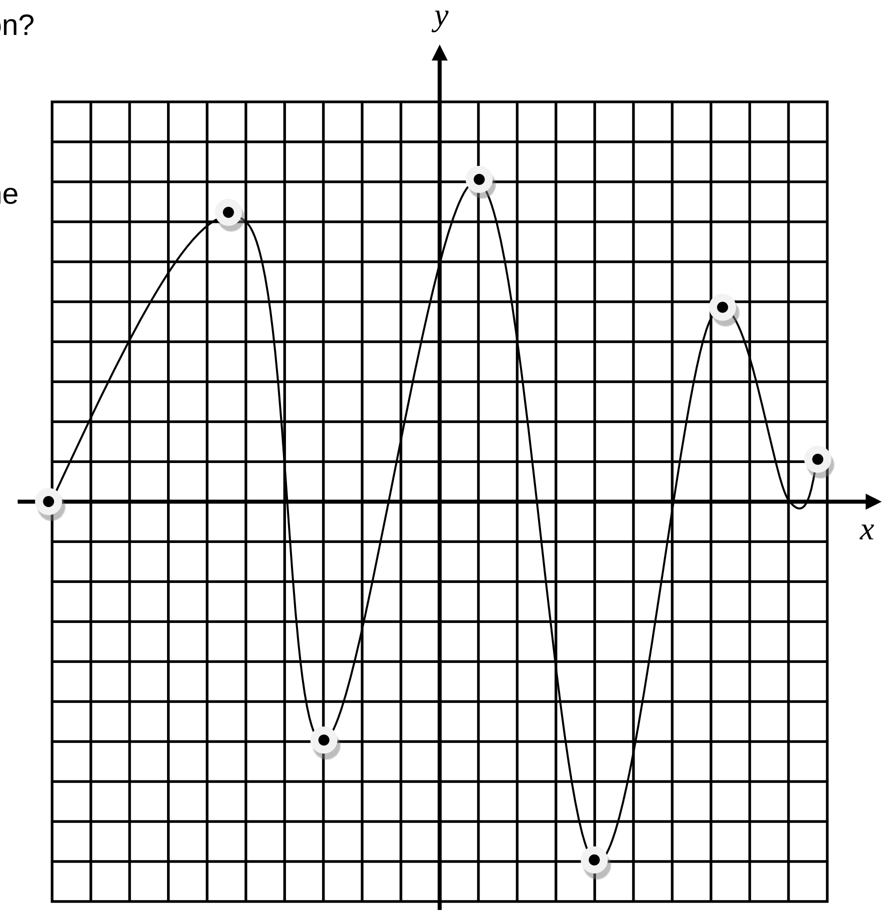
Increasing: $0 < x < 3$

Decreasing: $-3 < x < 0$





3. (a) What is the domain of the function?



(b) How many turning points does the graph of the function have?

(c) What is the maximum value of the function?

(d) What is the x-coordinate of the point where the function is a maximum?

(e) What is the minimum value of the function?

(f) What is the x-coordinate of the point where the function is a minimum?

(g) What is the range of the function?

(h) On which interval below is the graph of the function <u>decreasing</u>?

(1) $-9 < x < -5.2$

(3) $1 < x < 4$

(2) $4 < x < 6$

(4) $-3 < x < 4$

4. Explain why the graph of a function can have at most one y-intercept.

Graphing Functions
Intro to Geometry

AIM: → To graph linear, quadratic, exponential, square root, and absolute value functions

Do Now

Consider the function $y = \frac{1}{\sqrt{x+4}}$.

(a) What value of *x* is not allowed because it creates a zero in the denominator?

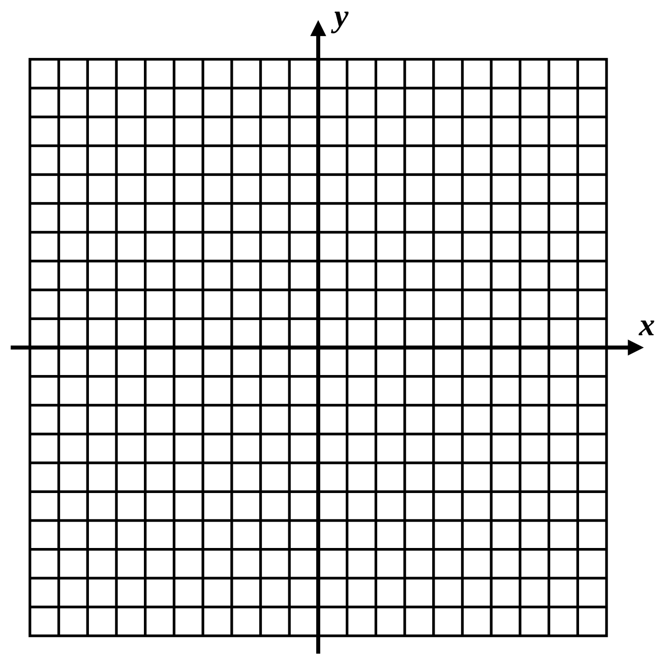
(b) What values of *x* are not allowed because they create a negative radicand?

(c) State the domain of the function.

Today we will review how to graph five special functions: linear, quadratic, exponential, square root, and absolute value. Remember to make a table of values to create points for the graph of the function as well as label the axes and graph.

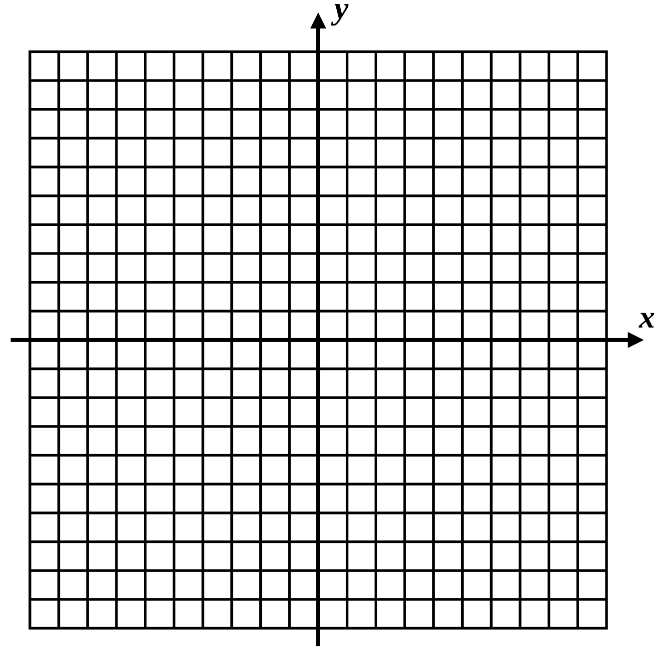
Linear Function

Graph the function $y = -2x + 7$ for all values of x on the interval $-1 \le x \le 5$.



Quadratic Function

Graph the function $y = x^2 - 2x - 2$ for all values of x on the interval $-2 \le x \le 4$.

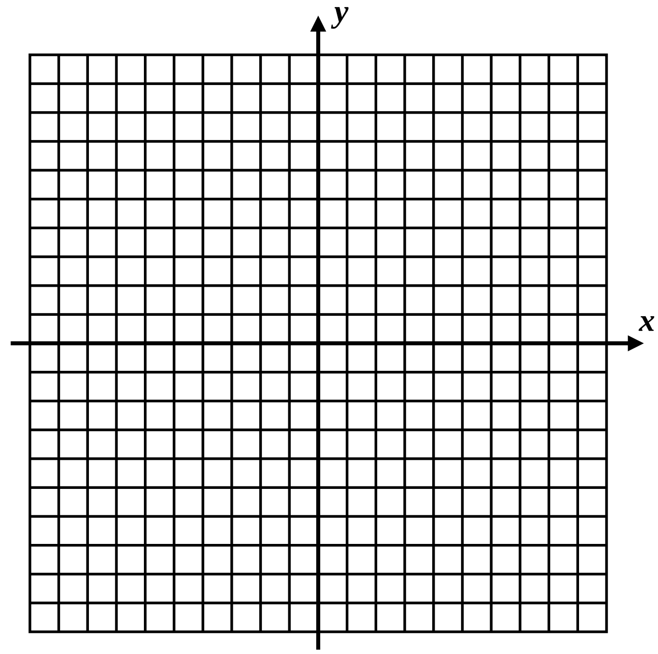


(a) What is the minimum of this function?

(b) What is the range of this function?

Exponential Function

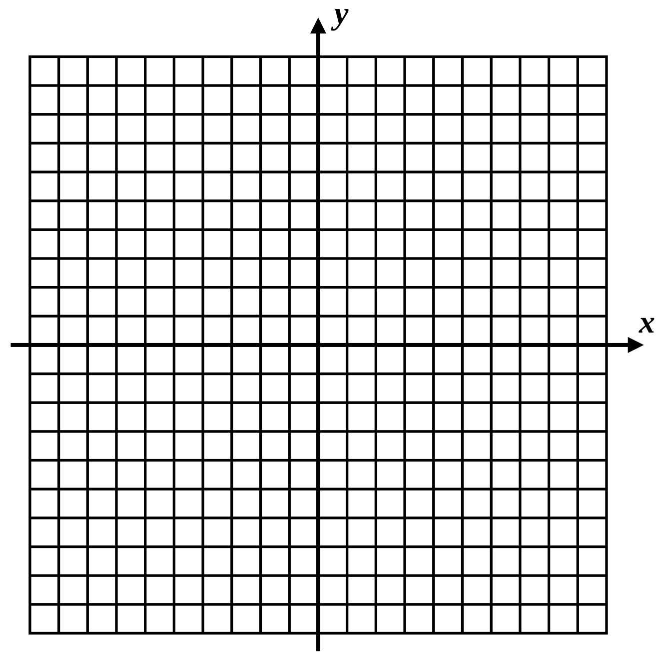
Graph the function $y = 2^x - 4$ for all values of x on the interval $-2 \le x \le 4$.



(a) Is this function increasing over its entire domain or decreasing over its entire domain?

Square Root Function

Graph the function $y = \sqrt{x+3} - 2$ for all values of x on the interval $-3 \le x \le 6$.

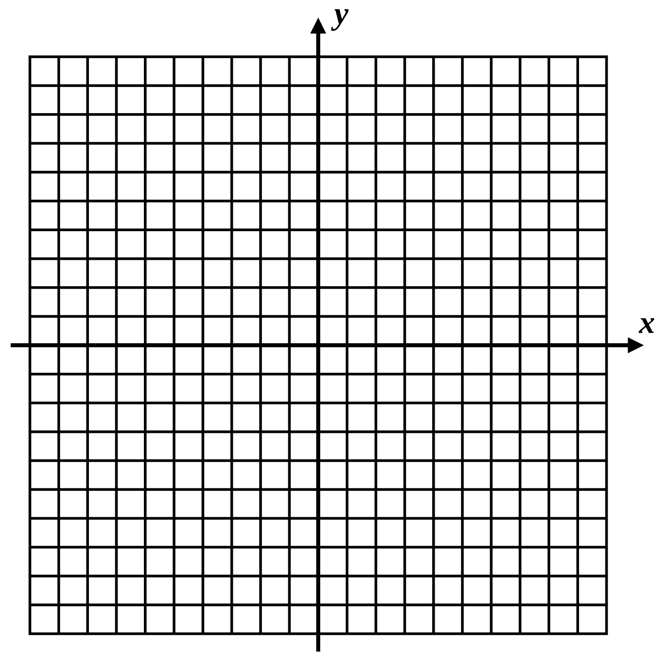


(a) Why is $x = -4$ not in the domain of this function?

(b) Why is $y = -5$ not in the range of this function?

Absolute Value Function

Graph the function $y = |x-3| - 5$ for all values of x on the interval $-1 \le x \le 7$.

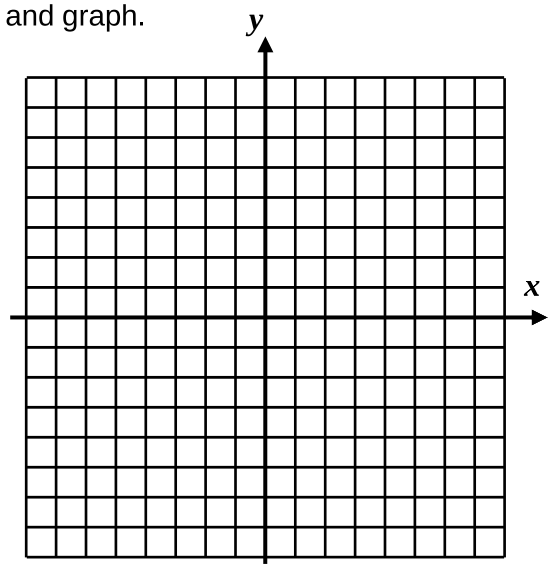


(a) What is the range of this function?

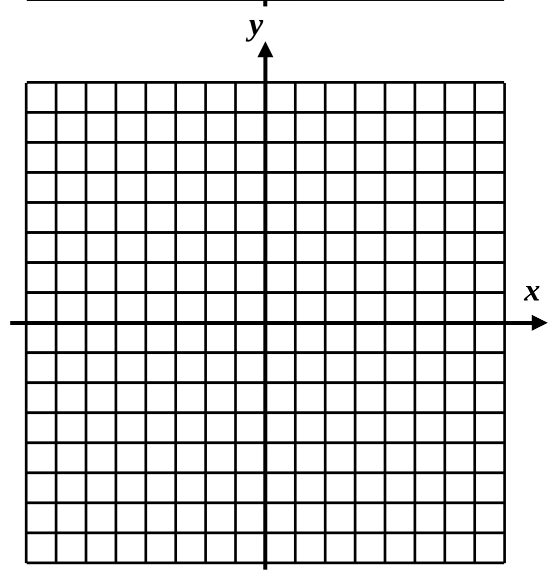
Graphing Functions- HOMEWORK

Directions: For each of the following, graph the function for the stated interval and state the range. Remember to show a table and to label the axes and graph.

1. $y = x - 2$ for $-4 \le x \le 5$

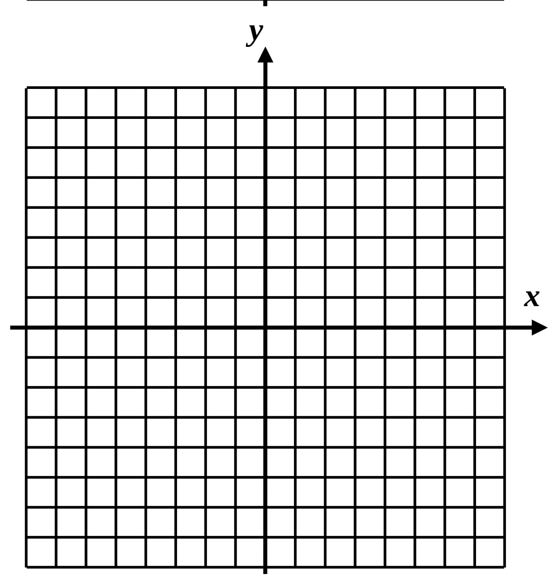


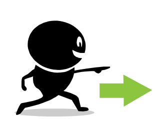
2. $y = x^2 + 2x - 1$ for $-4 \le x \le 2$



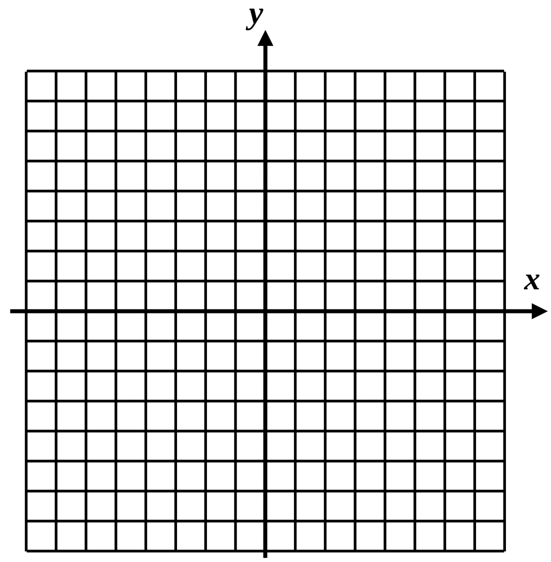
3. $y = 3^x - 5$ for $0 \le x \le 3$

Use an appropriate scale!

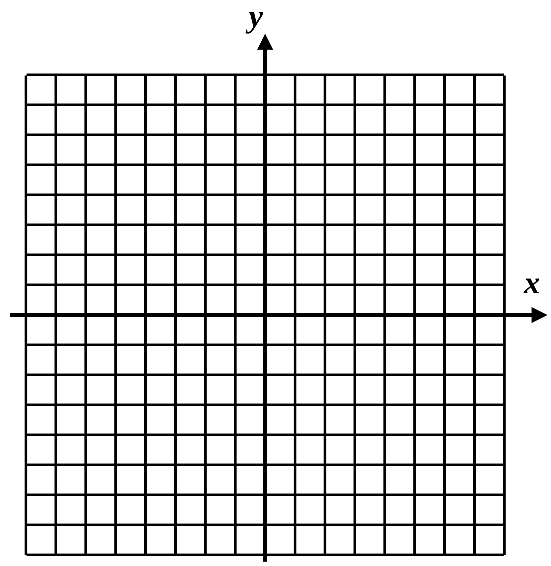




4. $y=\sqrt{x+2}+1$ for $-2 \le x \le 7$



5. $y=-|x+4|+6$ for $-7 \le x \le 1$



Review

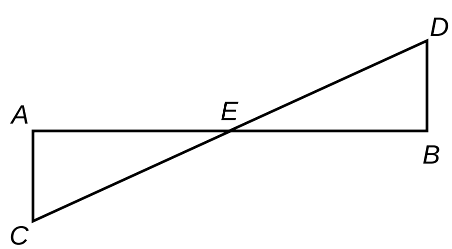
6. What is $\sqrt{72}$ expressed in simplest radical form? __________

(1) $2\sqrt{18}$ (2) $3\sqrt{8}$ (3) $6\sqrt{2}$ (4) $8\sqrt{3}$

7. Given: $\overline{AB} \perp \overline{BD}$, $\overline{AB} \perp \overline{AC}$

E is the midpoint of $\overline{AB}$ and $\overline{CD}$

Prove: $\Delta CAE \cong \Delta DBE$



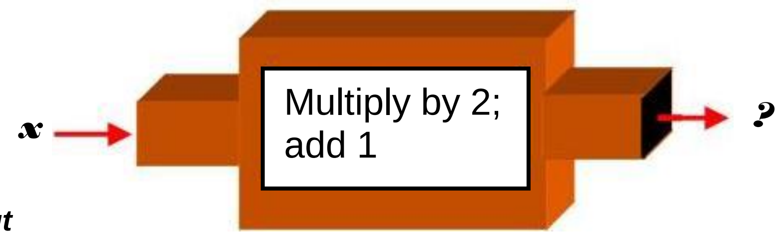
Function Notation
Intro to Geometry

AIM: → ***To recall the definition of function***
→ ***To introduce function notation***
→ ***To evaluate functions written in function notation***

A function is a rule because it assigns to each input, or *x*-value, exactly *one* output, or *y*-value.

Do Now

Think of a function like a machine—you give the machine raw materials as an input and you receive a finished product as the output.
For each input below, can you predict what the output will be?



Input		***Output***
3	→ →	________
-4	→ →	________
n	→ →	________

Function Notation

It is customary to denote functions by lower case letters (e.g. f, g, h). We use f most often to represent particular functions.

Ex. 1 Given the descriptions below, state each function (or rule) as an algebraic equation.
Use the first one as a guide

(a) "subtract 3, then multiply by 2"
- Calling our unknown *x*, then the equation would be $(x-3)2 = f(x)$
- Can you think of another way to state this equation?

(b) "multiply by 2, then add 7"

(c) "subtract 1, then take the square root"

(d) "multiply by -2, add 5, then take the square root"

Breaking Down the Rule ➡

$f(x)=4x+1$ means for any input x, we multiply that input by 4 and then add 1 in order to obtain the output $f(x)$. For example, $f(3)=13$ because $4(3)+1=13$.

<u>Ex. 2</u> Using $f(x)=4x+1$, find the value of the following:

(a) $f(2)$ (b) $f(0)$ (c) $f(-3)$ (d) $f(a)$

<u>Ex. 3</u> If function g is defined by $g(x)=x^2+x$, then find the value of each of the following:

(a) $g(5)$ (b) $g(2)$ (c) $g(-2)$ (d) $g(0)$

> By now you should realize that in the context of functions, $f(x)$ *<u>does not mean</u>* "variable f times variable x." $f(x)$ represents the <u>output</u> resulting by applying rule f to whatever number x represents.

<u>Ex. 4</u> If $f(2)=9$, then which ordered pair lies on the graph of the function f ?

→ *Hint: Think about which number is the input (x) and which is the output $f(x)$*

(1) $(9,2)$ (2) $(2,9)$ (3) $(2,2)$ (4) $(9,9)$

<u>Ex. 5</u> If $h(x)=4x+12$, then complete the given table.

x	$h(x)$
1	
	24
0.25	

Function Notation- HOMEWORK
Intro to Geometry

1. If $f(x) = |x-2|$, then find the value of each of the following.

(a) $f(4)$ (b) $f(-4)$ (c) $f(2)$

2. If $g(x) = -x^2 + x$, then find the value of each of the following.

(a) $g(3)$ (b) $g(-3)$

3. If $f(x) = 3-4x$ then complete the given table.

x	$h(x)$
-2	
	35
4	

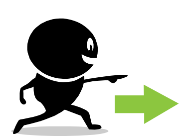
4. (a) If $f(x) = 3\sqrt{x-4}$, express each of the following values in *simplest radical form*.

(i) $f(22)$ (ii) $f(54)$

(b) What is the domain of the function defined above?

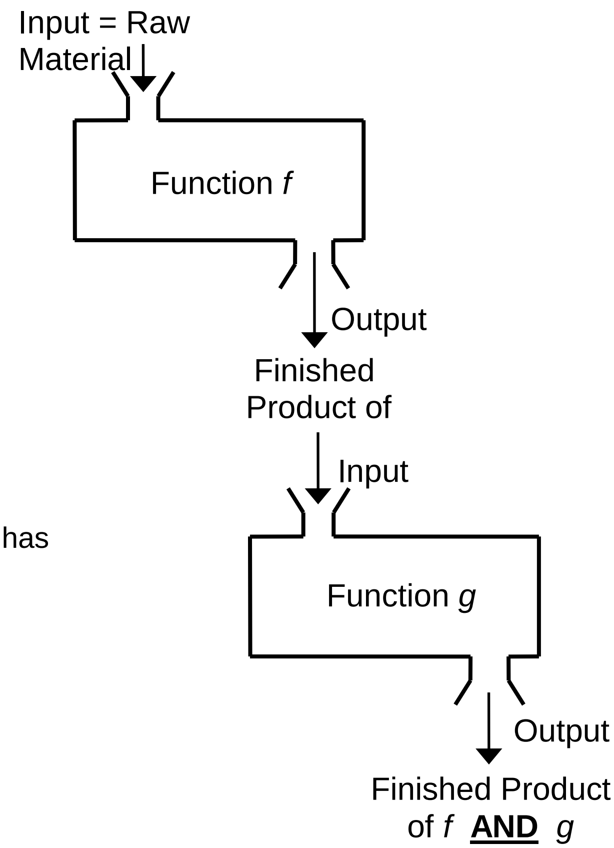
5. If $g(3) = -7$, then which point must lie on the graph of the function g?

(1) $(3,-7)$ (2) $(7,-3)$ (3) $(-7,3)$ (4) $(0,0)$



Composition of Functions
Intro to Geometry

AIM: → *To define a composition of functions*
→ *To evaluate compositions of functions*



Yesterday, we referred to a function as a "machine" that performed certain changes on inputs to produce corresponding outputs.

Suppose that we have two such machines, one called *f*, the other called *g*. Imagine setting up these two machines in an assembly line, so that as soon as the finished product comes out of machine *f*, it goes into machine *g*.

The result of the assembly line is a finished product that has been shaped by function *f* AND function *g*.

Ex. 1 Given $f(x) = 2x + 7$ and $g(x) = x^2 - 4$, determine the output using the above diagram for the inputs of: (a) $x = 0$ (b) $x = -3$ (c) $x = 2$

(a) **First:** Input goes into function f and is evaluated: $f(0) = 2(0) + 7 = 7$

Second: Output of function f is input for function g: $g(7) = (7)^2 - 4$

Third: Evaluate function g:
$$\begin{aligned} g(7) &= (7)^2 - 4 \\ &= 49 - 4 \\ &= 45 \end{aligned}$$

Your turn:

(b)

(c)

Composition Notation

There are two common ways to show the composition of functions.

Method 1

$g\left(f\left(x\right)\right)$

In this method, you evaluate the composition by working from the *inside to the outside*. x is evaluated in f first, and then in g.

Method 2

$\left(g \circ f\right)\left(x\right)$

In this method, you evaluate the composition by working from *right to left*. x is evaluated in f first, and then in g.

Ex. 2 (Method 1) Given that $f\left(x\right) = 2x - 3$ and $g\left(x\right) = 2^x$, evaluate:

(a) $f\left(g\left(2\right)\right)$

(b) $g\left(f\left(2\right)\right)$

Ex. 3 (Method 2) Given that $f\left(x\right) = x - 2$ and $g\left(x\right) = \sqrt{x}$, evaluate:

(a) $\left(g \circ f\right)\left(11\right)$

(b) $\left(f \circ g\right)\left(11\right)$

Notice that which function you use first makes a difference in the final answer you obtain from the composition. **Always pay attention to the order of the composition.**

Ex. 4 Given that $f\left(x\right) = x^2 - 1$ and $g\left(x\right) = \left|x\right| - 6$, evaluate each of the following compositions.

(a) $\left(f \circ g\right)\left(-2\right)$

(b) $g\left(f\left(3\right)\right)$

(c) $\left(g \circ f\right)\left(0\right)$

Composition of Functions- HOMEWORK
Intro to Geometry

1. Given $f(x)=x^2$, $g(x)=3x$, and $h(x)=x-1$, evaluate each of the following compositions.

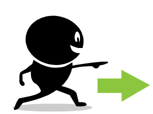
(a) $(f\circ g)(1)$ (b) $(h\circ f)(3)$ (c) $(f\circ h)(3)$

(d) $f(h(-3))$ (e) $g(h(-2))$ (f) $f\left(h\left(-\frac{1}{4}\right)\right)$

2. A Samsung Blu-Ray player is on sale for 25% off of the regular price. On top of that, you have a coupon for $10 off the regular price. Is it better to have the $10 taken off before the 25% off or after the 25% off?
(Think about how this is the composition of two functions and show calculations for both final prices.)

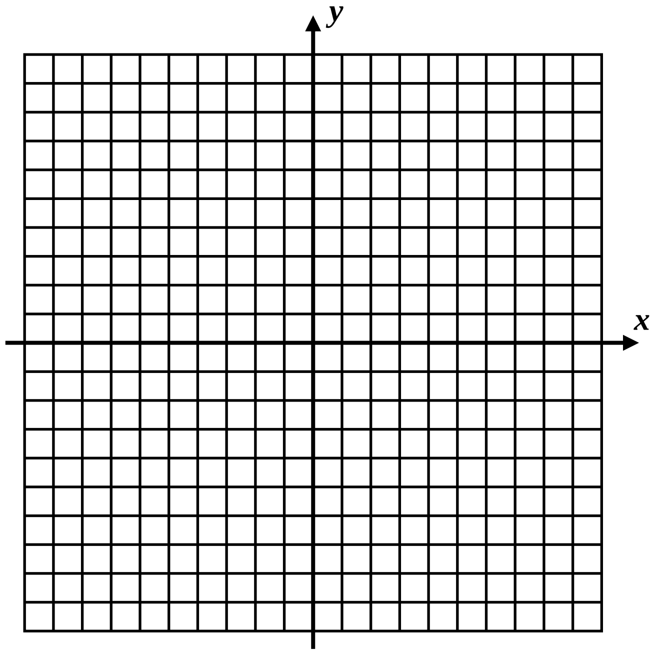
3. Given $h(x)=\sqrt{x-1}$ and $g(x)=-2x+6$, what is the output of $g(h(5))$?

(1) $\sqrt{5}$ (2) $2\sqrt{3}$ (3) -2 (4) 2



Review

4. If the domain is $\{x : -3 \le x \le 3\}$, graph $f(x) = x^2 + 2$ and state the range.



Range:___________________________________