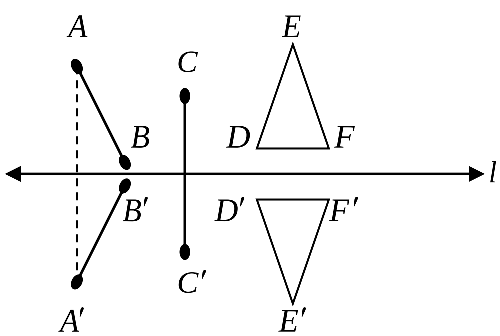
Name: ______________________ Date: _____________ **UNIT 8 BOOK**

LINE REFLECTIONS

Intro to Geometry

AIM: → To identify important vocabulary: line reflection, image, and pre-image
→ To develop rules for line reflections in the *x*-axis, *y*-axis, line *y* = *x*, and line *y*=-*x*
→ To find images by reflecting over horizontal and vertical lines

A ***line reflection*** is a "flip" across a given line such that the distance the **image** is from the line is equal to the original, or **pre-image**. Reflections preserve shape and size, but not orientation.

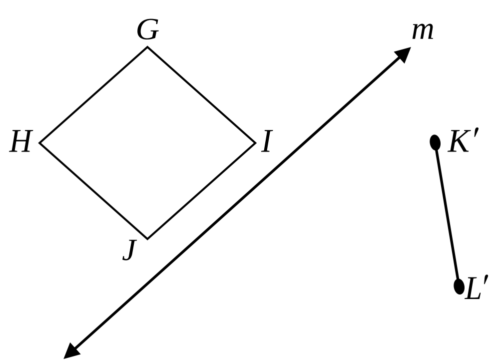


A' is called the **image** of point A after a reflection over line *l*.

Similarly, A is called the **pre-image** of A'.

Notice that the line *l*
1. Is perpendicular to AA', BB', CC', etc.
2. Bisects each segment (AA', BB', CC', etc.)

Ex. 1



(1) Sketch the *image* of the square to the left.

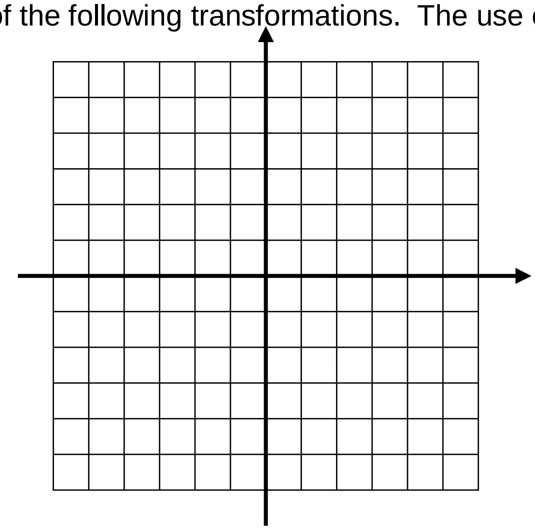
(2) Sketch the *pre-image* of the line segment under r_m (reflection over the line *m*)

Line Reflection Notation: In order to specify a line reflection, we only need to refer to what line we are reflecting about. The following notation specifies reflections about various lines:

$$r_{x-axis},\ r_{y-axis},\ r_{y=x},\ r_{y=2},\ r_{x=-1},\ \text{etc.}$$

Notice reflections use the lower-case *r*.

<u>Ex. 2</u> Given the point (-4, 3) find its **image** under each of the following transformations. The use of the graph paper at the right is optional.



(a) r_{x-axis}

(b) r_{y-axis}

(c) $r_{y=x}$

(d) $r_{y=-x}$

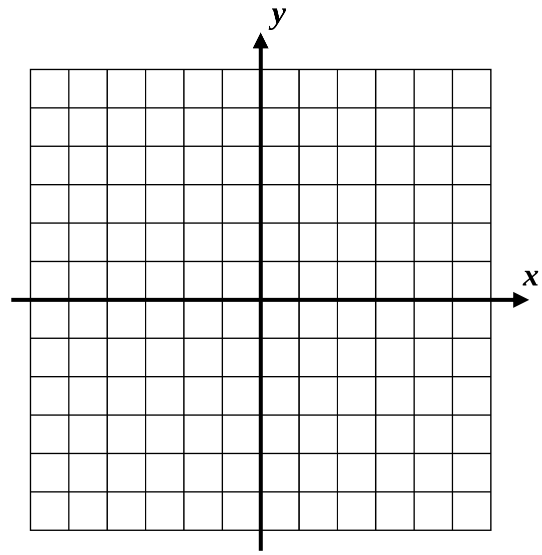
(e) $r_{x=2}$

Let's look at specific examples in order to develop rules for certain line reflections.

<u>Reflections in the x-axis</u>

Triangle *ABC* has vertices $A(-2,3), B(4,-1),$ and $C(-3,-4)$.

(a) Plot the triangle on the coordinate axes.
(b) Plot and state the coordinates of the image of *ABC* under a reflection in the *x*-axis.



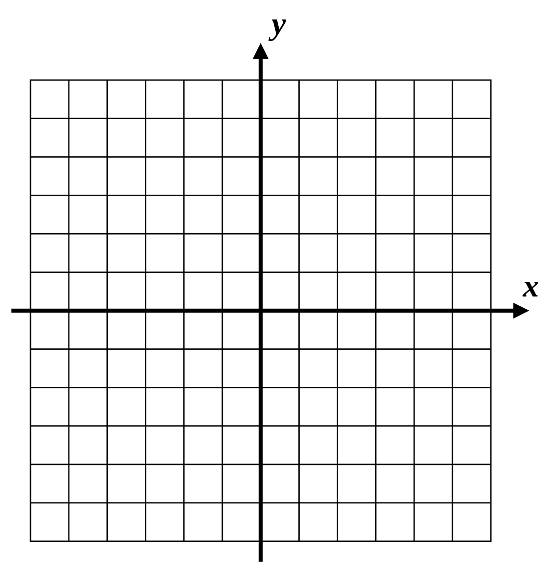
Reflections in the *x*-axis

$A(x,y) \rightarrow$

<u>Reflections in the y-axis</u>

Triangle *ABC* has vertices $A(1,4), B(4,3),$ and $C(-2,-2)$.

(a) Plot the triangle on the coordinate axes.
(b) Plot and state the coordinates of the image of *ABC* under a reflection in the *y*-axis.



Reflections in the *y*-axis

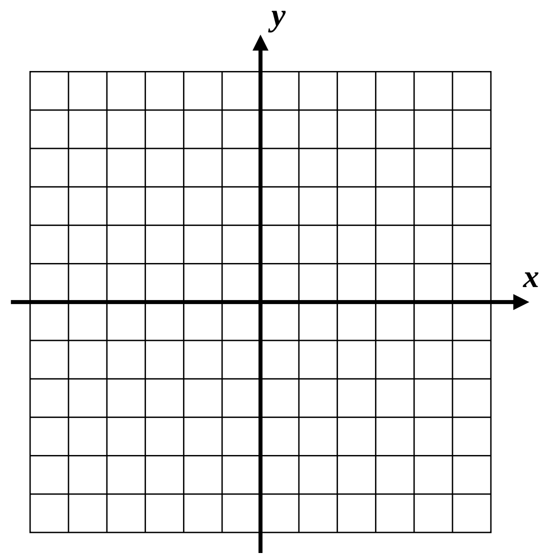
$A(x,y) \rightarrow$

Reflections in the Line $y = x$

Triangle *ABC* has vertices $A(-1,4), B(3,1)$, and $C(2,-2)$.

(a) Plot the triangle on the coordinate axes.

(b) Plot and state the coordinates of the image of *ABC* under a reflection in the line $y = x$.



Reflections in $y = x$

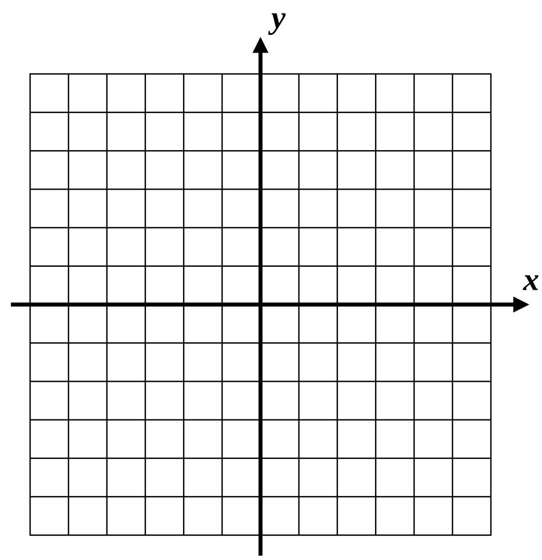
$A(x, y) \rightarrow$

Reflections in the Line $y = -x$

Triangle *ABC* has vertices $A(0,5), B(-4,1)$, and $C(3,3)$.

(a) Plot the triangle on the coordinate axes.

(b) Plot and state the coordinates of the image of *ABC* under a reflection in the line $y = -x$.



Reflections in $y = -x$

$A(x, y) \rightarrow$

Reflections Over Other Lines

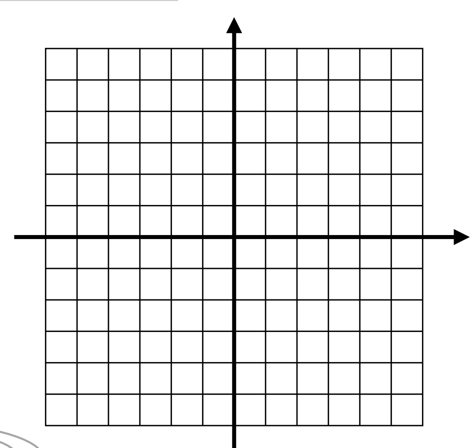
We will not use any general rule for these types of line reflections and will do these graphically.

Recall: $x = \#$ (Vertical Line) $y = \#$ (Horizontal Line)

Ex. 3 Find the image of each point.

(a) $r_{x=3} : (1,4)$

(b) $r_{y=-2} : (-3,2)$

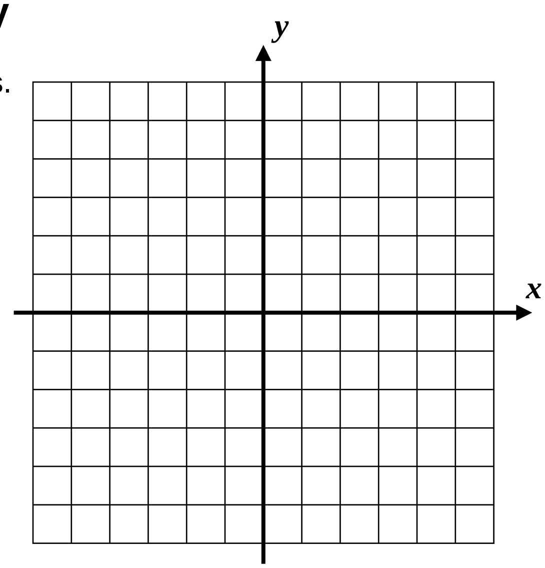


Line Reflections HOMEWORK
Intro to Geometry

1. Find the image of each point under a reflection in the *x*-axis. Plot both the original point and its image on the set of axes provided. (Do **not** connect!)

   (a) $A(3,2)$ (b) $B(-2,-4)$ (c) $C(1,5)$

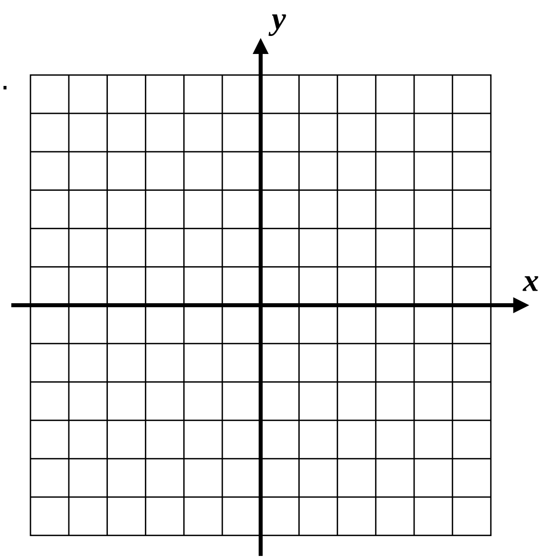
   (b) $D(4,-5)$ (e) $E(-6,1)$



2. Find the image of each point under a reflection in the *y*-axis. Plot both the original point and its image on the set of axes provided. (Do **not** connect!)

   (a) $A(6,2)$ (b) $B(-2,-5)$ (c) $C(1,5)$

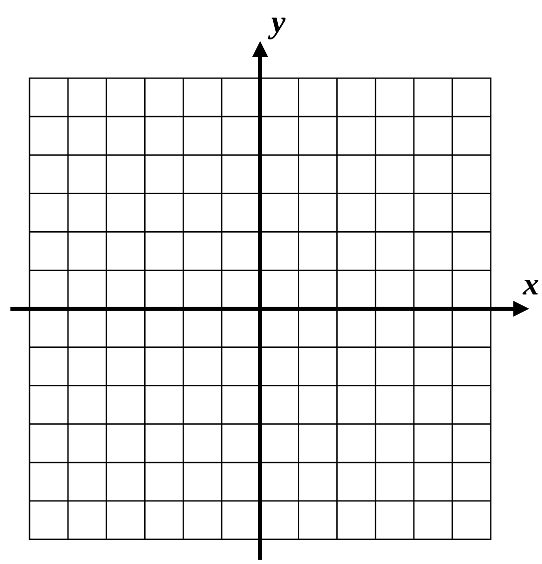
   (b) $D(3,-6)$ (e) $E(-6,4)$



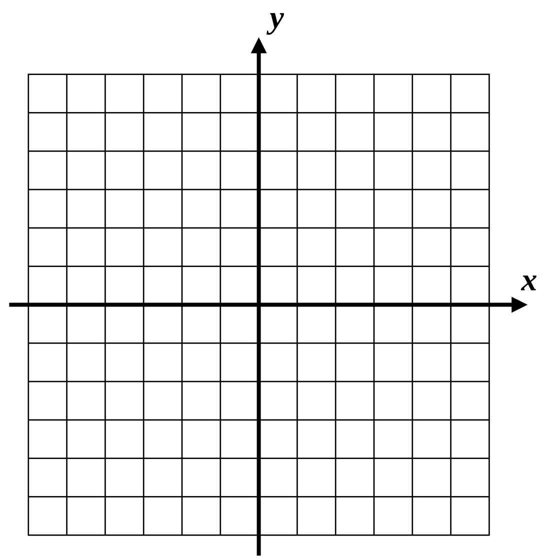
3. Find the image of each point under a reflection in the line $y = x$. Plot both the original point and its image on the set of axes provided. (Do **not** connect!)

   (a) $A(3,2)$ (b) $B(-2,-4)$ (c) $C(1,6)$

   (b) $D(4,-5)$ (e) $E(-6,1)$



4. Find the image of each point under a reflection in the line $y = -x$. Plot both the original point and its image on the set of axes provided. (Do **not** connect!)



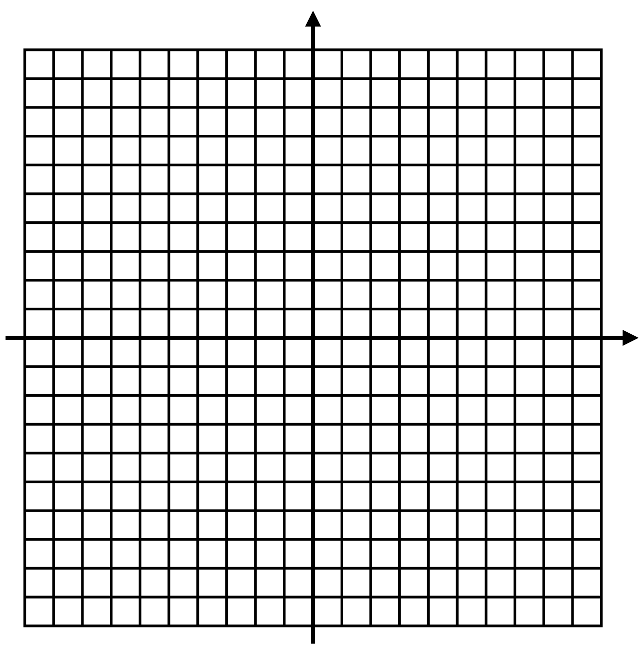
(a) $A(4,1)$ (b) $B(-4,-4)$ (c) $C(1,3)$

(b) $D(2,-5)$ (e) $E(-6,3)$

5. Point $R(-2,5)$ was reflected to point $R'(2,5)$. What is the image of $F(5,3)$ under the same type of reflection?

6. Find the image of the point $(-4,-2)$ under a reflection in each of the following vertical and horizontal lines. Recall that vertical lines have *x* = # equations and horizontal lines have *y* = # equations.

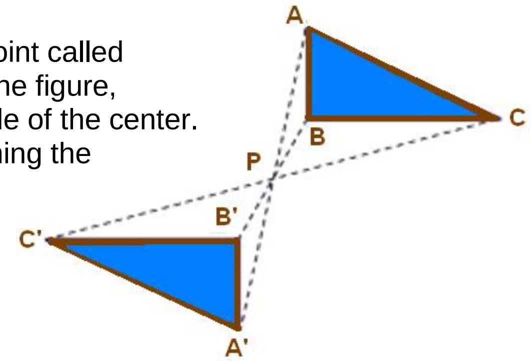
(a) $r_{x=-2}$ (b) $r_{y=3}$ (c) $r_{x=3}$ (d) $r_{y=-4}$



Point Reflections
Intro to Geometry

AIM: → To define point reflection
→ To find the point of reflection 1) *graphically* and 2) *algebraically*

A ***point reflection*** exists when a figure is built around a single point called the center of the figure, or point of reflection. For every point in the figure, there is another point found directly opposite of it on the other side of the center. The point of reflection is actually the midpoint of the segment joining the point with its image.



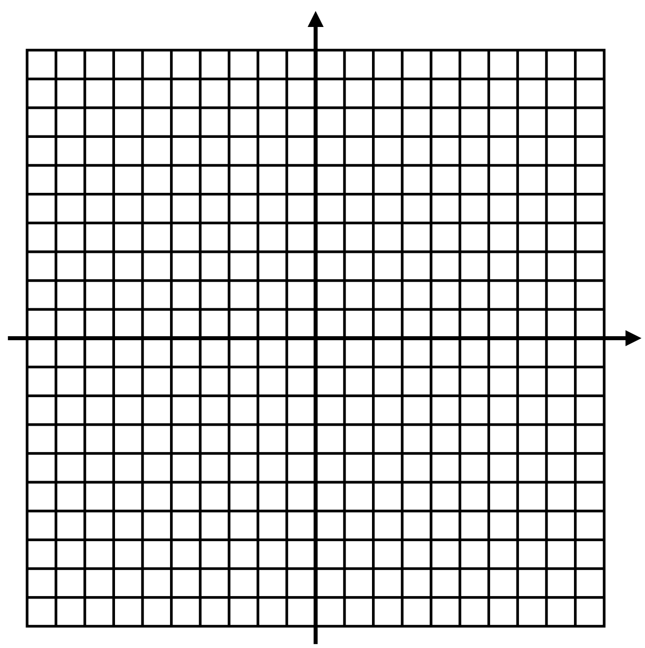
Point reflections preserve shape, size, *and* orientation.

Given the image and pre-image of a point, we can find the point of reflection in two ways:

Method 1: Graphically

- Starting from the pre-image, count up/down from the pre-image to the image and left/right from the pre-image to the image – note this distance
- Starting from the pre-image again, count up/down, left/right **half** as far to find the *point of reflection*

Ex. 1 Find the point of reflection given $A(-5,1)$ and $A'(3,7)$.



Ex. 2 Find the point of reflection given $G(-3,-4)$ and $G'(1,2)$.

Ex. 3 Find the *image* given $O(2,-4)$ and the point of reflection $(4,-1)$.

Method 2: Algebraically

Since the point of reflection is actually the midpoint of the pre-image and the image, we can find it using the following adaptation of the midpoint formula:

$$\text{Point of Reflection} = \left(\frac{x_{pre-image} + x_{image}}{2}, \frac{y_{pre-image} + y_{image}}{2} \right)$$

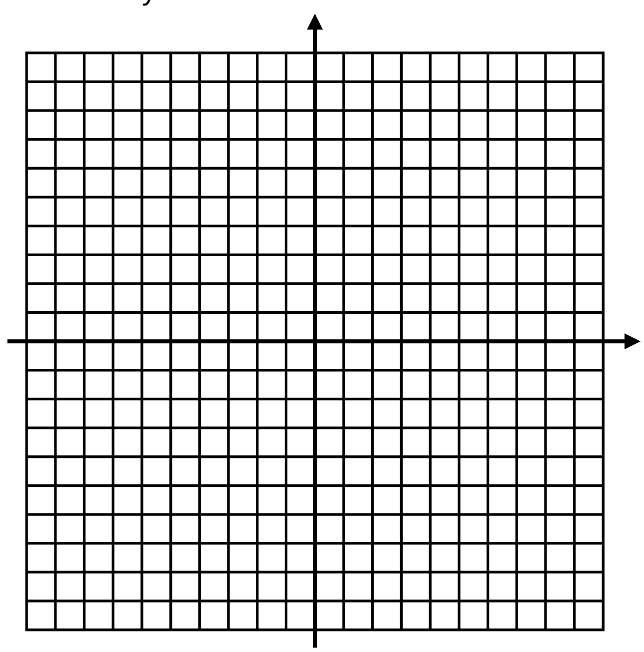
Ex. 4 Find the point of reflection given the pre-image $C(3,1)$ and the image $C'(-1,-1)$.

Ex. 5 Find the point of reflection given the pre-image $R(-2,-5)$ and the image $R'(2,5)$.

CLASS PRACTICE

Find the point of reflection given the following sets of points. You may use either method.

(a) $Z(-3,2)$ and $Z'(-5,-6)$



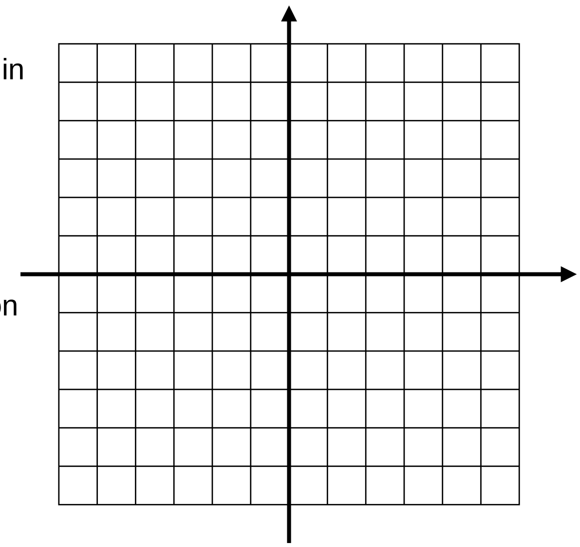
(b) $Y(0,0)$ and $Y'(-8,-6)$

(c) $X(3,0)$ and $X'(8,6)$

(d) $W\left(\frac{1}{2},4\right)$ and $W'(3,8)$

Actually reflecting a point in another point is also relatively easy. You count the number of spaces horizontally and vertically to the point of reflection, then count the same number of spaces (horizontally and vertically) past that point.

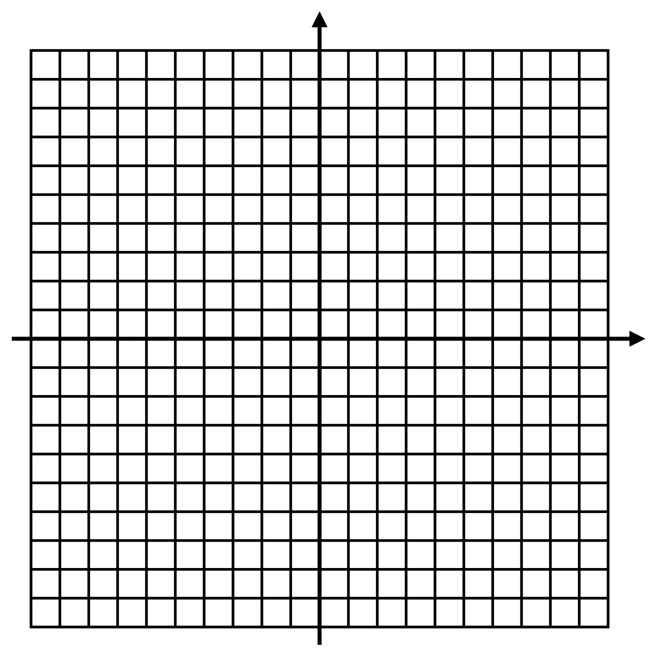
Point A has coordinates (3,4). Find the image of A after a reflection in the point (-1, -2).



Point Q has coordinates (4, -3). Find the image of Q after a reflection in the Origin (0,0).

Class Practice

Reflect each of the following points in a given point.



a) A (-8, -6) in B (-1, -3) ________________

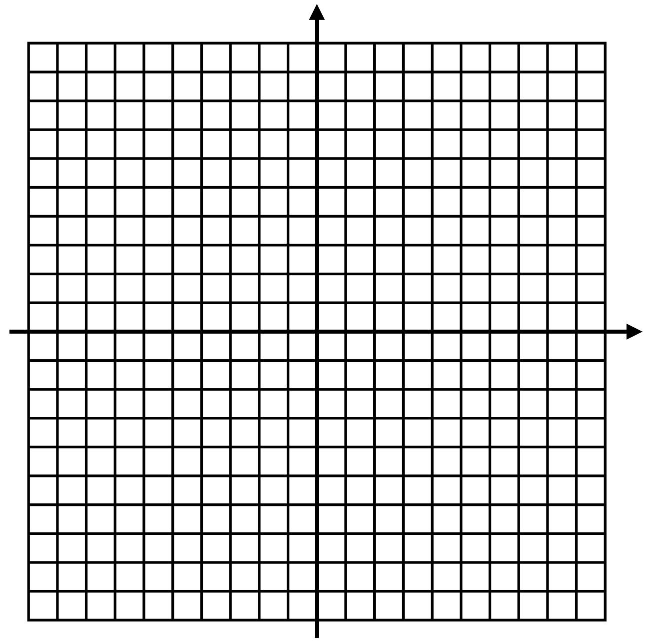
b) C (5, -7) in D (7, 1) ________________

c) E (-9, -4) in F (-5, -6) ________________

d) G (4,8) in H (6, 1) ________________

Point Reflections HOMEWORK
Intro to Geometry

1. Find the point of reflection given the following points. You may use a graphical or algebraic method.



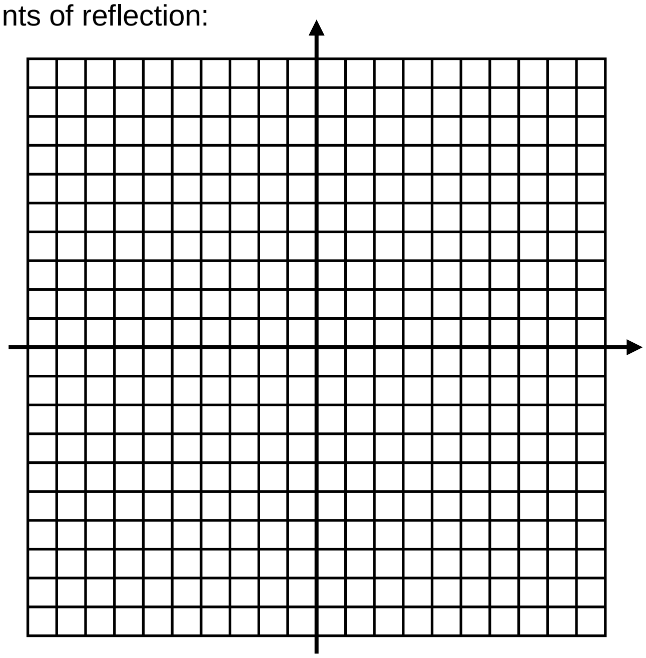
(a) $A(1,5)$ and $A'(7,9)$

(b) $B(-2,3)$ and $B'(4,1)$

(c) $C(-4,-6)$ and $C'(3,-3)$

(d) $D(1,-3)$ and $D'(2,-1)$

2. Find E' given the point $E(5,-4)$ over the following points of reflection:
(i.e. Find E' using ***each*** point of reflection below)



(a) $(1,-6)$ (b) $(0,2)$

(c) $(-2,-2)$ (d) $(3,1.5)$

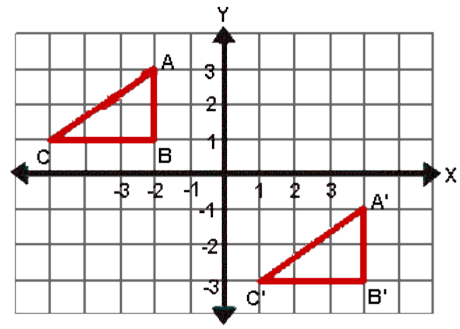
Translations
Intro to Geometry

AIM: → **To define translation**
→ **To find the coordinates of an image after a translation**
→ **To identify a translation, $T_{h,k}$, that maps a pre-image to an image**

A ***translation*** is a transformation that "slides" a point or object a given distance and direction. Translations are denoted as follows:

$$T_{h,k}$$

This translation (T) would move each point h units to the left/right and k units to the up/down.



*What do you think is the translation, T, for the triangle *ABC* above?

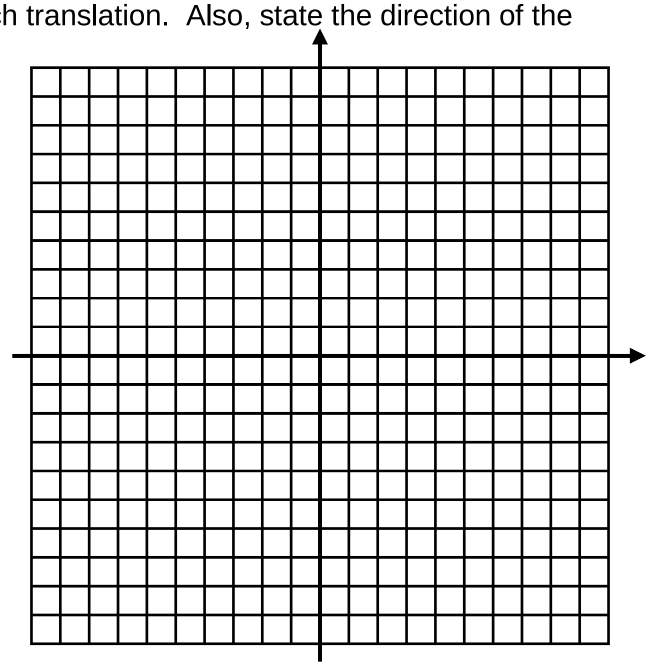
Ex. 1 Find the coordinates of the point $(-2,4)$ after each translation. Also, state the direction of the movement. (Left/Right and Up/Down)

a) $T_{3,2}$

b) $T_{0,2}$

c) $T_{-2,5}$

d) $T_{-3,-4}$



Do you notice any relation between the numbers given in the translation and the coordinates after each move?

We can develop a general rule for calculating the coordinates of a point after a translation:

$$T_{h,k}:(x,y)\rightarrow$$

Ex. 2 Determine the coordinates of each point after the given translation.

a) $T_{2,-6}:(3,10)$ b) $T_{-20,10}:(40,17)$ c) $T_{-50,-100}:(3,52)$

Ex. 3 Name the translation, in the form $T_{h,k}$, that maps the first point onto the second.

a) $(-5,9)\rightarrow(-2,10)$ b) $(1,8)\rightarrow(-4,6)$ c) $(3,-5)\rightarrow(7,2)$

Ex. 4 Triangle *ABC* has vertices $A(-4,-1), B(-2,3),$ and $C(3,-3)$.

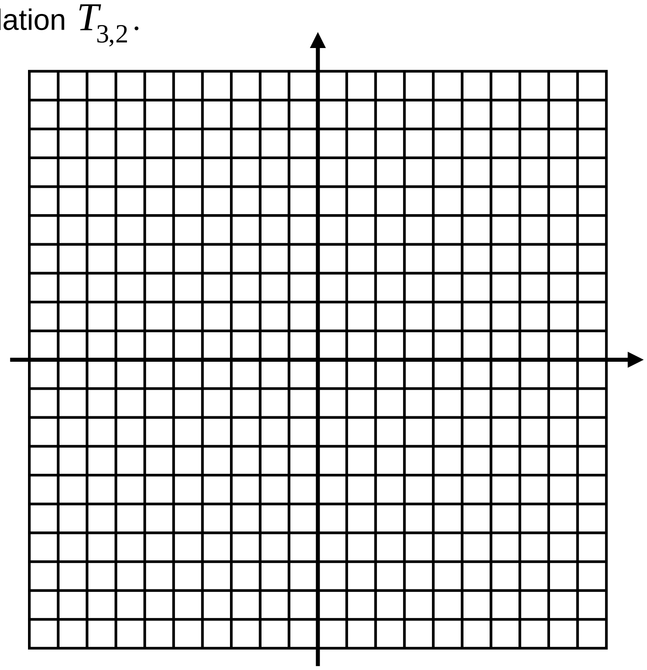
a) Plot the original triangle.

b) Plot the image of the triangle under the translation $T_{3,2}$.

$A(-4,-1)\rightarrow$

$B(-2,3)\rightarrow$

$C(3,-3)\rightarrow$



Ex. 5 Find the pre-image of each point for the given transformation.

a) $(8,-2)$ under $T_{-3,5}$ b) $(0,11)$ under $T_{4,-7}$ c) $(-6,-7)$ under $T_{8,-6}$

Translations HOMEWORK
Intro to Geometry

1. Find the image of the point (-6, -2) under each of the following translations.

(a) $T_{5,-4}$ (b) $T_{-2,7}$ (c) $T_{0,10}$ (d) $T_{5,-5}$

2. The point *A*(-3, 4) undergoes a translation such that its image has the coordinates given in each of the following problems. State the translation, in the form of $T_{h,k}$, that gives each image.

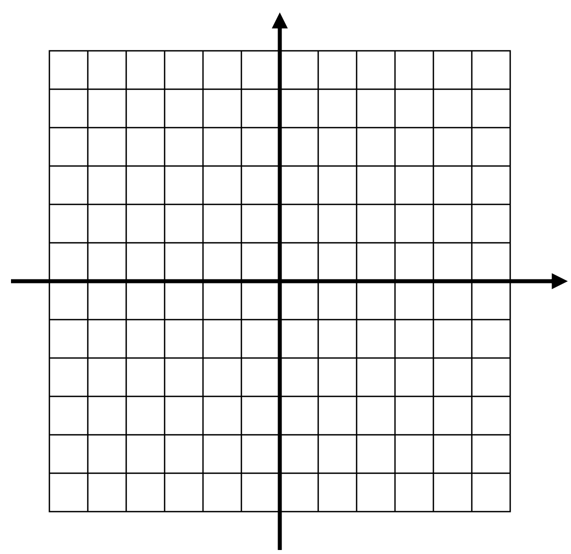
(a) $A'(2,1)$ (b) $A'(-10,0)$ (c) $A'(-6,8)$ (d) $A'(0,7)$

3. Find the pre-image of each point for the given transformation.

a) $(10,-19)$ under $T_{-7,20}$ b) $(-12,12)$ under $T_{19,-11}$ c) $(-5,-20)$ under $T_{18,-16}$

4. The point (-6, 8) is translated to the point (3, 2). What is the image of the point (4, -1) under the same translation?

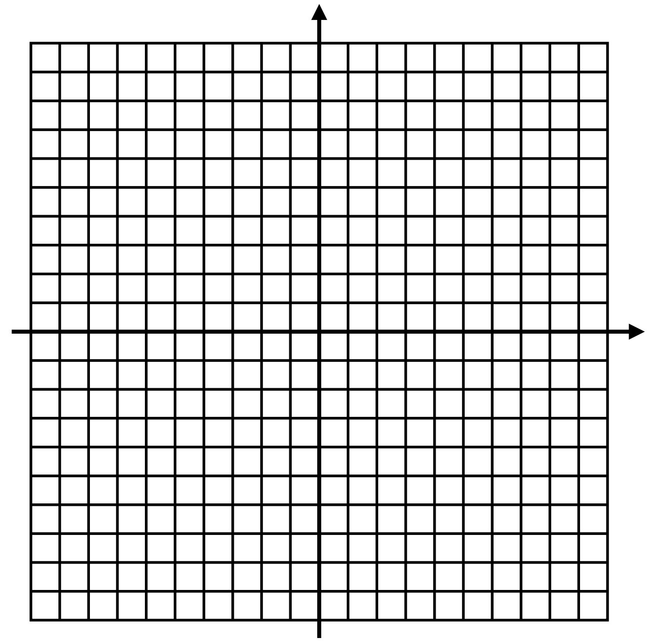
5. The point (7, -5) is translated to the point (2, -8). What is the image of the point (3, 10) under the same translation?



6. (a) On the axis to the right sketch the triangle with vertices $A(1,3)$, $B(2,-2)$, $C(5,2)$

(b) Determine the coordinates of the vertices of the triangle under the transformation $T_{-6,-2}$. Graph and label these points A', B', C'.

REVIEW (from yesterday)



1. Find the image of $(5,6)$ after a reflection in the x-axis.

2. Find the image of $(-3,2)$ after a reflection in the y-axis.

3. Find the image of $(1,-5)$ after a reflection in the line $y=x$.

4. Find the image of $(-7,3)$ after a reflection in the origin (0,0).

5. Find the image of $(-2,-5)$ after a reflection in the line $y=-x$.

Dilations Review & Practice
Intro to Geometry

AIM: → *To recall the definition of dilation*
→ *To find images under dilations and to determine the dilation constant for various dilations*

Recall:

Dilation: A transformation that stretches or shrinks an original figure to produce an image that is the same shape, but different in size.

*We will only be studying dilations with the origin as the center of dilation.

Since a dilated figure does not retain size, it **does not preserve distance** (does not keep the same distance between sides, vertices, points, etc. as original figure)

A dilation with scale factor *k* can be written as:

$$D_k(x, y) = (kx, ky)$$

If scale factor *k* has a value that is ***greater than 1***, then the image is an ***expansion***.

If the scale factor *k* has a value that is **between 0 and 1**, then the image is a **contraction**.

*A scale factor of 1 just creates $\cong$ figures.

Ex. 1 Find the image of (-6, 12) under each of the following dilations.

(a) D_2 (b) $D_{\frac{1}{3}}$ (c) $D_{\frac{1}{6}}$

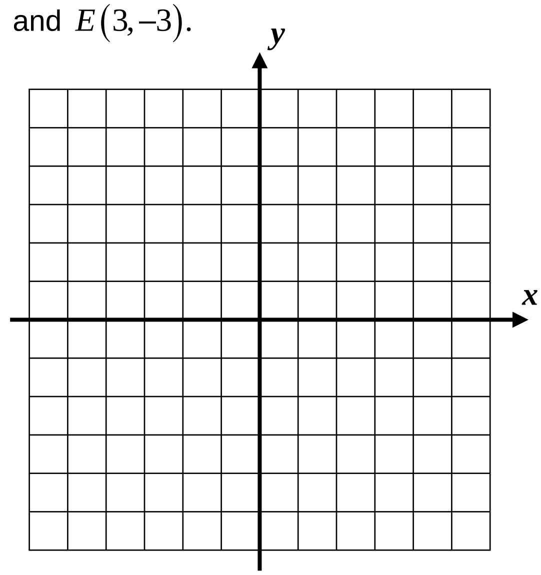
Ex. 2 In each of the following problems, the point *A* has been mapped to its image *A'* under a dilation. Determine the scale factor *k* for each dilation.

(a) $A(2,5) \to A'(6,15)$ (b) $A(8,4) \to A'(2,1)$ (c) $A(6,-4) \to A'(9,-6)$

Ex. 3 The image of $(8,4)$ is $(4,2)$ after a dilation. Find the **pre-image** of $(1,-6)$ under the same dilation.

Ex. 4 For pentagon *ABCDE*, $A(0,0), B(3,3), C(6,3), D(6,-3)$, and $E(3,-3)$.

(a) Plot and label the points on the grid.



(b) Find the image of pentagon *ABCDE* under the dilation $D_{\frac{1}{3}}$.

A'(,) *B*'(,) *C*'(,)

D'(,) *E*'(,)

(c) Find the area of pentagon *ABCDE*.

(d) Find the area of the image *A*'*B*'*C*'*D*'*E*''.

(e) How much did the area increase by? Did this result surprise you?

Dilation Class Practice:

1. Find the image of (-2, 8) under a dilation $D_{\frac{1}{2}}$. ____________
2. If $A(8,12) \to A'(2,3)$ under a dilation, what is the scale factor? ______________
3. The image of $(5,1)$ is $(15,3)$ after a dilation. Find the **pre-image** of $(3,-6)$ under the same dilation.

Dilations Review & Practice - HOMEWORK
Intro to Geometry

1. Which mapping represents a dilation? 1. __________

(1) $(x, y) \to (y, x)$ (3) $(x, y) \to (2x, 2y)$

(2) $(x, y) \to (x+2, y+2)$ (4) $(x, y) \to (-y, -x)$

2. Find the image of $(3, -2)$ under the dilation D_2.

3. If the dilation $D_k(-2, 4)$ equals $(1, -2)$, the scale factor *k* is equal to 3. __________

(1) -2 (2) 2 (3) $-\frac{1}{2}$ (4) $\frac{1}{2}$

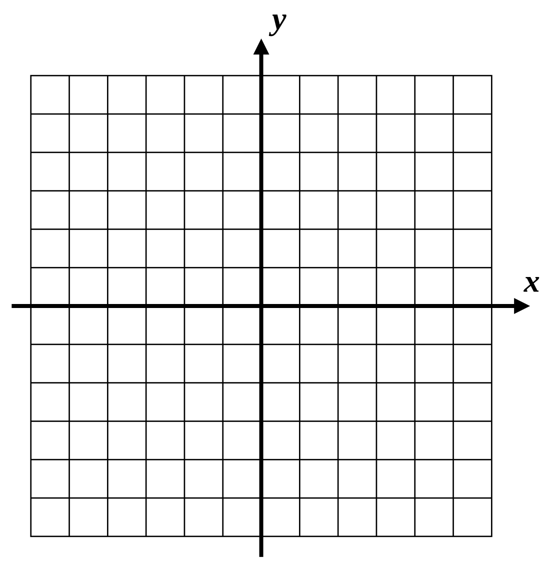
4. For triangle *ABC*, $A(-2,-2), B(1,-1), C(0,2)$.

(a) Plot and label the points on the grid.

(b) Find the image of triangle *ABC* under the dilation D_2.

A'(,) *B*'(,) *C*'(,)

(c) Find the area of the image *A*'*B*'*C*'.



REVIEW

5. If the lengths of two sides of a triangle are 4 and 10, which of the following could be the length of the third side?

(1) 14 (2) 6 (3) 8 (4) 16

6. Which one of the following sets of numbers may represent the lengths of the sides of a right triangle?

(1) $\{7,8,10\}$ (2) $\{5,5,10\}$ (3) $\{5,12,13\}$ (4) $\{4,5,6\}$

Glide Reflections: A Composition of Transformations
Intro to Geometry

AIM: → *To describe a composition of transformations*
→ *To define glide reflection*
→ *To find images under glide reflections*

Do Now

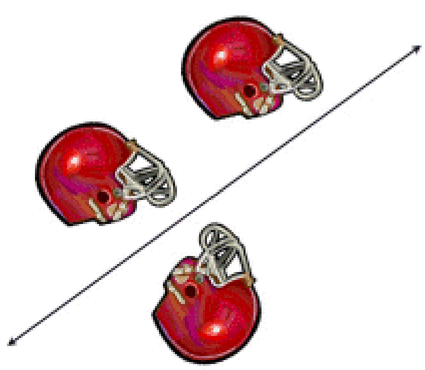
For any given point, (x, y), write the rule in either symbols or words, for the reflection in each line, and then reflect the point (2, 3) in each line:

a.) r_{x-axis}

c.) $r_{y=x}$

b.) r_{y-axis}

d.) $r_{y=-x}$



When two or more transformations are combined to make a new transformation, the result is called a **composition of transformations**.

A **glide reflection** is the composition of a ***translation and a reflection***. This transformation is commutative.

In a glide reflection, the direction of the translation must be ***parallel*** to the line over which you reflect.

In a composition of functions, the first transformation produces an image and the second transformation is performed on that image to produce a new one. We use an open circle to denote a composition of functions.

Be careful--a composition of functions is read **right to left:**

$$r_{y-axis} \circ T_{0,2}$$

SECOND **FIRST**

Translation of $(x+0, y+2)$,
then a reflection over the y-axis

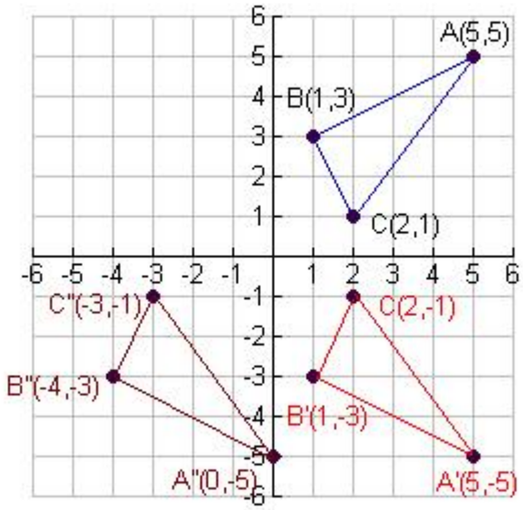
Ex. 1 Which of the following is *not* an example of a glide reflection?

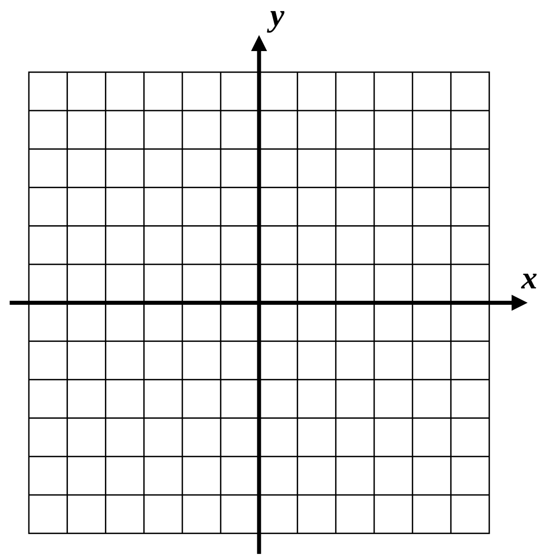
(1) $T_{-2,5} \circ r_{x-axis}$ (2) $r_{x-axis} \circ T_{0,5}$ (3) $r_{y-axis} \circ T_{0,-4}$ (4) $r_{y=5} \circ r_{y=x}$

Ex. 2 Give the coordinates of the image of (-2, 3) under each of the following glide reflections:

(a) $r_{y-axis} \circ T_{0,-5}$ (b) $T_{-6,0} \circ r_{x-axis}$ (c) $T_{3,3} \circ r_{y=x}$ (d) $r_{y=-x} \circ T_{4,-4}$

Ex. 3 Given the graph below, is triangle *A''B''C''* a glide reflection of triangle *ABC*? If so, come up with the rule.





Ex. 4 (a) On the grid to the right, plot the triangle with vertices $A(1,3),\ B(2,-2),\ C(5,2)$.

(b) Determine the coordinates of the vertices of the triangle under the transformation $T_{0,-4} \circ r_{y-axis}$. Graph and label these points A', B', C'.

A'(,) *B'*(,) *C'*(,)

Glide Reflections: A Composition of Transformations

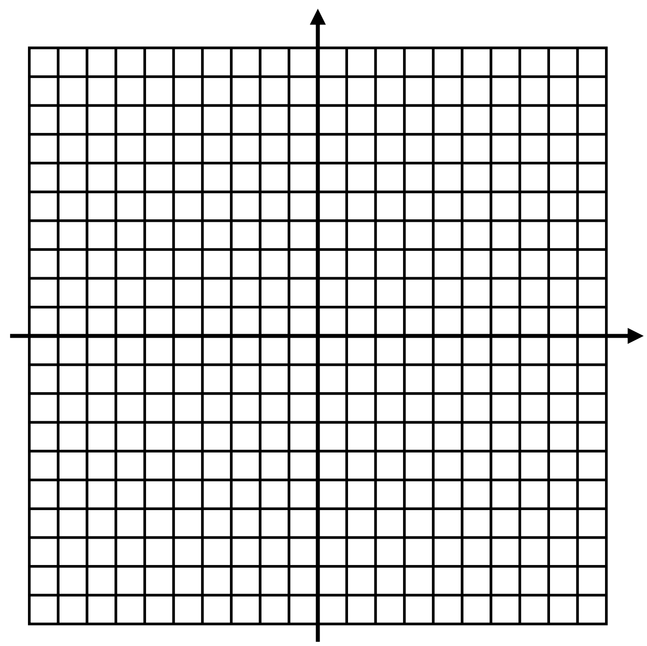
Class Practice/Homework

1. A triangle has vertices $A(3,2), B(4,1)$, and $C(4,3)$. What are the coordinates of the triangle under glide reflection $T_{0,1} \circ r_{y-axis}$?

2. Given triangle *ABC*: $A(1,4), B(3,7), C(5,1)$.
Graph and label the following composition:

$$T_{-5,0} \circ r_{x-axis}$$

A'(,) *B*'(,) *C*'(,)



3. Give the coordinates of the image of (-3, 7) under the glide reflection $T_{2,-2} \circ r_{y=-x}$

4. Give the coordinates of the image of (4, 0) under the glide reflection $r_{x-axis} \circ T_{2,0}$

5. Give the coordinates of the image of (11, -2) under the glide reflection $r_{y=x} \circ T_{-5,-5}$

6. Give the coordinates of the image of (-6, 17) under the glide reflection $T_{0,1} \circ r_{y-axis}$

Rotations
Intro to Geometry

<u>AIM</u>: → **To define rotation**
→ **To find the images of points and figures under clockwise and counterclockwise rotation** $(90^\circ, 180^\circ, 270^\circ, 360^\circ)$

Rotation

A transformation in which all of the points in the original figure rotate, or turn, through the same angle, about the same center, in the same direction.

A rotation is about a single point called the **center of rotation**.

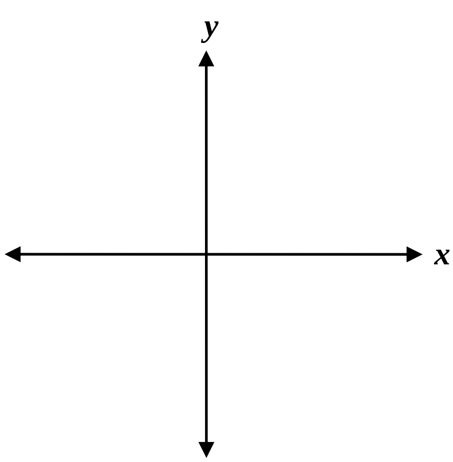
Although rotations can be about any point, we will focus on those that take place about the origin $(0,0)$.

A rotation can be either ***clockwise*** or ***counterclockwise***.

➡<u>Negative angle</u> of rotation - turns the figure *clockwise*
➡<u>Positive angle</u> of rotation - turns the figure *counterclockwise*

*If no direction is given, assume the direction of rotation is counterclockwise.

It is helpful to know the quadrants of a set of coordinate axes when rotating figures:



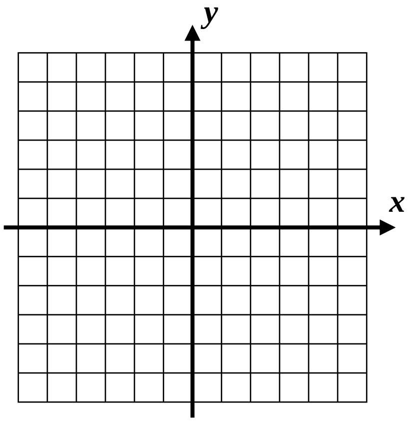
The notation used for a rotation is:
$R_{\text{center of rotation, angle of rotation}}$

Ex. $R_{0,90^\circ}$ stands for a rotation with center $(0,0)$ through an angle of 90° counterclockwise

*You may want to consider turning your paper to illustrate the rotations.

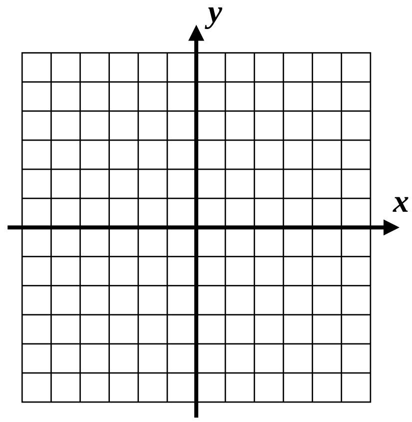
Let's look at $90^\circ, 180^\circ$, and 270° rotations: Plot the point (3, 2) to use as an illustration.

Rotation of 90°



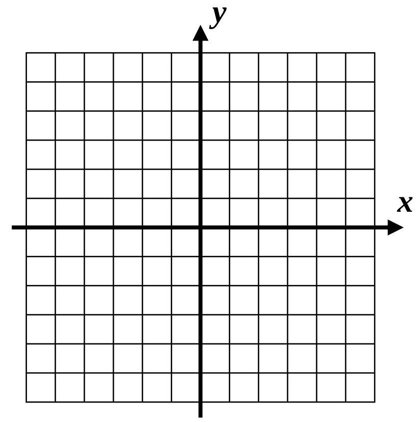
$R_{90^\circ}(x, y) = (\quad , \quad)$

Rotation of 180°



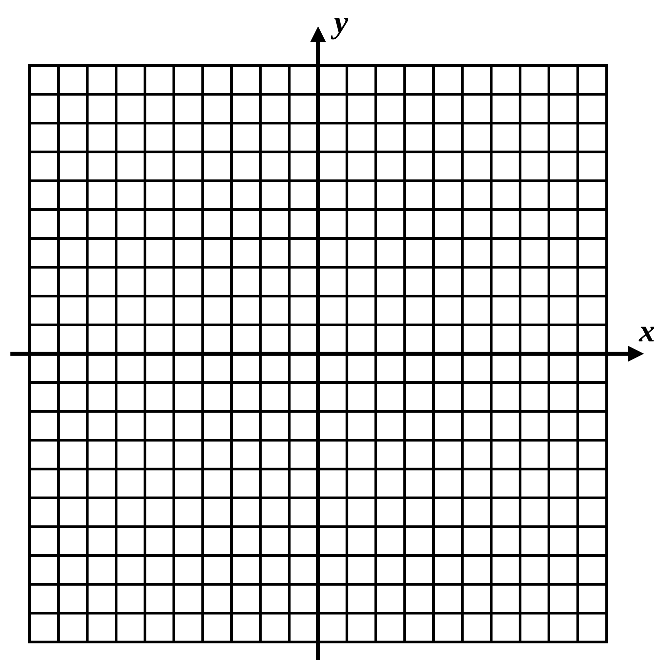
$R_{180^\circ}(x, y) = (\quad , \quad)$

Rotation of 270°



$R_{270^\circ}(x, y) = (\quad , \quad)$

<u>Ex. 1</u>



(a) What are the coordinates of M', the image of $M(2,4)$ after a counterclockwise rotation of 90° about the origin?

(b) What are the coordinates of B', the image of $B(-2,3)$ under $R_{0,90^\circ}$ in a clockwise direction?

(c) What are the coordinates of P', the image of $P(-5,-1)$ under $R_{0,180^\circ}$?

(d) What are the coordinates of Q', the image of $Q(2,-4)$ under $R_{0,270^\circ}$?

(e) What are the coordinates of H', the image of $H(-4,-7)$ after a clockwise rotation of 270° about the origin?

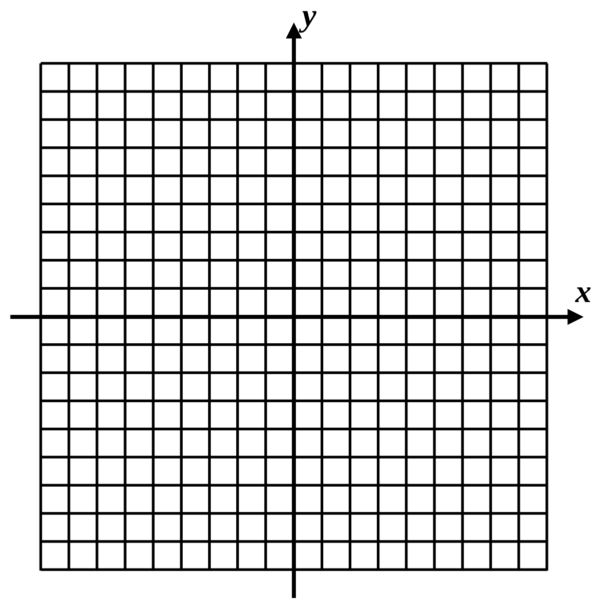
<u>Ex. 2</u> The transformation $R_{0,90^\circ}$ maps point $(5,3)$ onto the point whose coordinates are

(1) $(3,5)$ (2) $(3,-5)$ (3) $(-3,5)$ (4) $(5,-3)$

Ex. 3 Given points $W(-5,2), R(0,4),$ and $X(0,1),$ plot ΔWRX.

(a) Plot the image of ΔWRX under $R_{0,180^\circ}$.

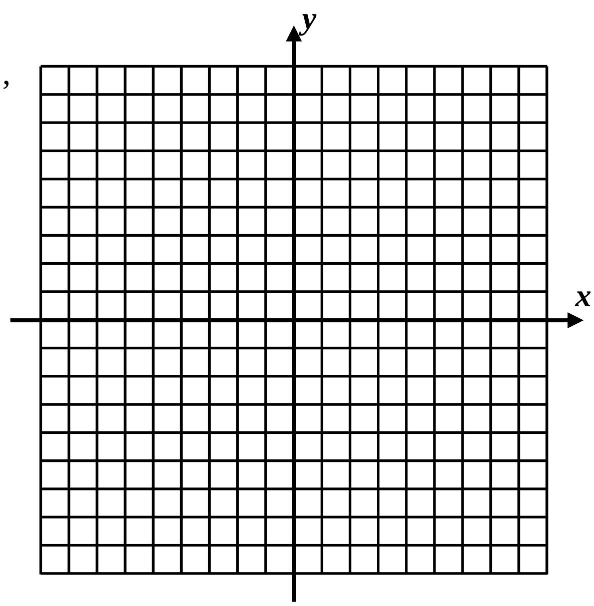
(b) List the coordinates of $\Delta W'R'X'$.



Ex. 4 Given points $F(-2,2), A(0,0), S(5,5),$ and $T(3,7),$ plot rectangle $FAST$.

(a) Plot the image of rectangle $FAST$ under $R_{0,270^\circ}$.

(b) List the coordinates of $F'A'S'T'$.



Ex. 5 Find the image of the point $(6,-2)$ under the rotation $R_{0,90^\circ}$ about the origin.

Rotations HOMEWORK
Intro to Geometry

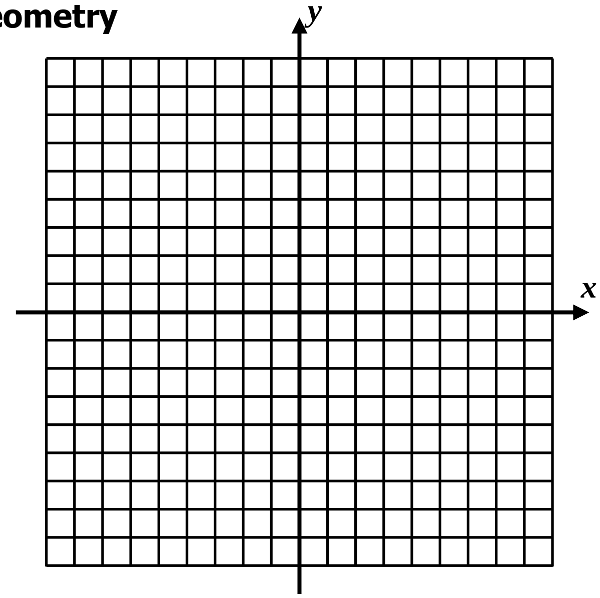
1. Given points $A(1,3)$, $B(5,1)$ and $C(1,1)$,
   (a) Plot ΔABC

   (b) Find the image of ΔABC under $R_{0,90°}$ and state the coordinates:

   A' _______________

   B' _______________

   C'_______________



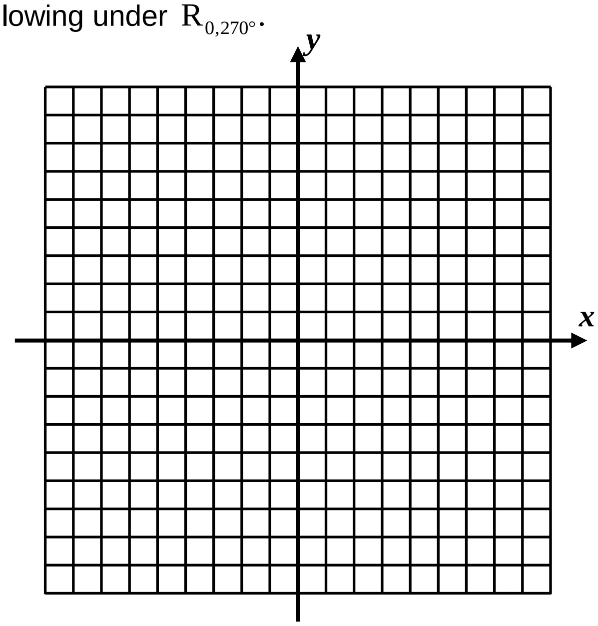
2. For ***each*** of the following, find the *image* under $R_{0,180°}$.

   (a) $(4,6)$ **(b)** $(-3,0)$ **(c)** $(-4,-2)$ **(d)** $\left(\frac{1}{3},\frac{1}{5}\right)$

3. Find the coordinates of the *image* of each of the following under $R_{0,270°}$.

   (a) $(6,-5)$ (c) $(10,-7)$

   (b) $(-8,-8)$



Review

4. Find the image of $(-4, 5)$ under each of the following.

(a) $R_{0,180°}$ **(b)** $R_{0,90°}$ **(c)** $T_{-5,6}$ **(d)** r_{x-axis}

(e) r_{y-axis} **(f)** $r_{y=x}$ **(g)** $R_{0,270°}$ **(h)** $T_{3,8}$

5. The measure of the vertex angle of an isosceles triangle is three times the measure of the base angle. Find the number of degrees in the measure of a base angle.

6. The measures of two supplementary angles are in the ratio 5:1. What is the measure of the *smaller* angle?

(1) 15° (2) 150° (3) 75° (4) 30°