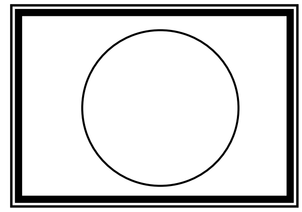
Name: ______________________________

UNIT 9 BOOK

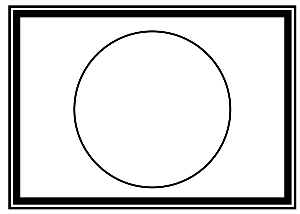
Lesson 1: Introduction to Circles
Intro to Geometry

Below are definitions we will need for our next unit: Circle Geometry.

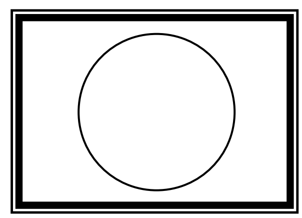
____________________________: set of all points in a plane that are a given distance from a given point



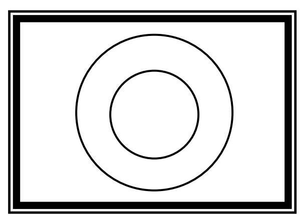
____________________________: the given fixed point



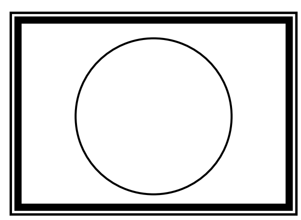
____________________________: a segment whose endpoints are the center and a point on the circumference of the circle



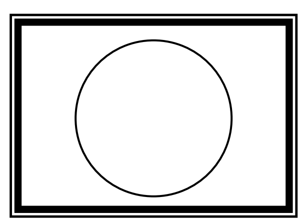
____________________________: circles with the same center



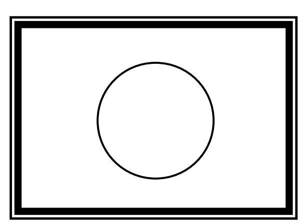
____________________________: a segment whose endpoints are on the circumference of a circle



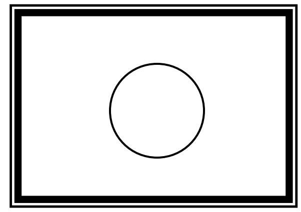
____________________________: the chord passing through the center of a circle



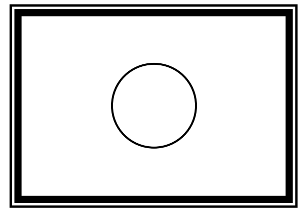
____________________________: a line that intersects a circle in two distinct points



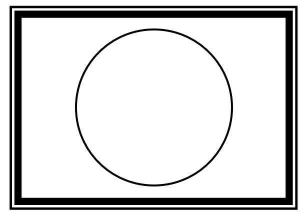
______________________________: a line that intersects a circle in exactly one point



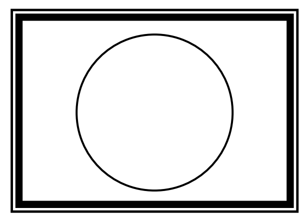
______________________________: polygon whose sides are all tangent to a circle



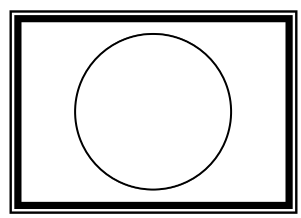
______________________________: polygon whose vertices all lie on the circumference of a circle



______________________________: (of a circle) is an angle whose vertex is at the center of the circle. This angle is equal to the arc it produces.

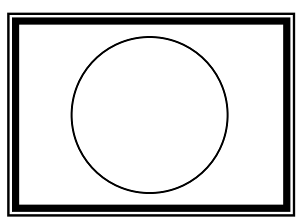


______________________________: consists of two points on a circle and all of the points on a circle needed to connect them by a single path

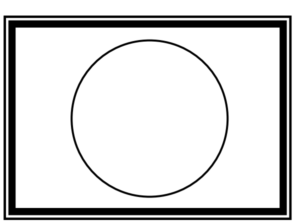


______________________________: arc that is equal to 180°

*Named with **2** points on the circle

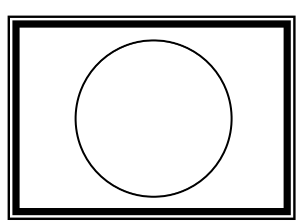


______________________________: arc with angle measure less than 180°



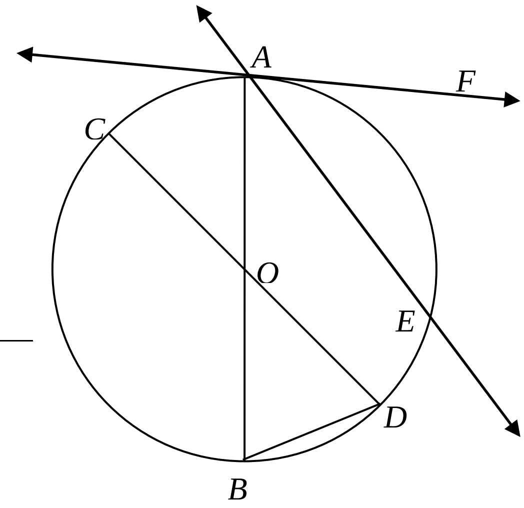
*Named with its endpoints

______________________________: arc with angle measure greater than 180°



*Named with **3** points on the circle

For exercises #1-10, refer to $\odot O$



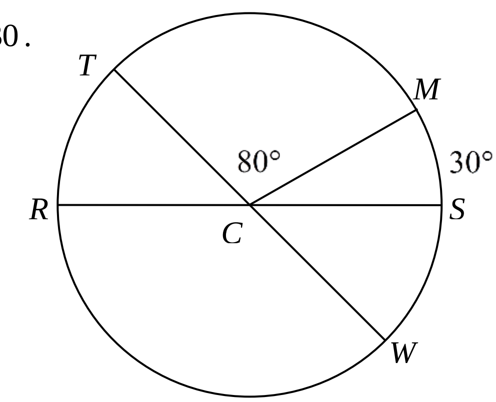
1. Name two chords in the diagram
2. Name two central angles that are supplementary
3. Name two diameters
4. If segment $\overline{DE}$ were drawn, it would be a ________________
5. Name a secant line
6. Name a tangent line
7. Name two major arcs

8. Name two minor arcs

9. Name the angle whose measure determines that of AC

10. How many radii are shown in the diagram?

For exercises #11-25, find the measure of each of the following
Given that $\overline{TW}$ and $\overline{RS}$ are diameters, $m\measuredangle TCM = 80°$ and $mMS = 30$.



11. mTM 12. $m\angle MCS$ 13. mTS

14. $m\angle SCW$ 15. $m\angle TCR$ 16. $m\angle RCW$

17. mRW 18. $mRWM$ 19. $mWRT$ 20. $mWRM$

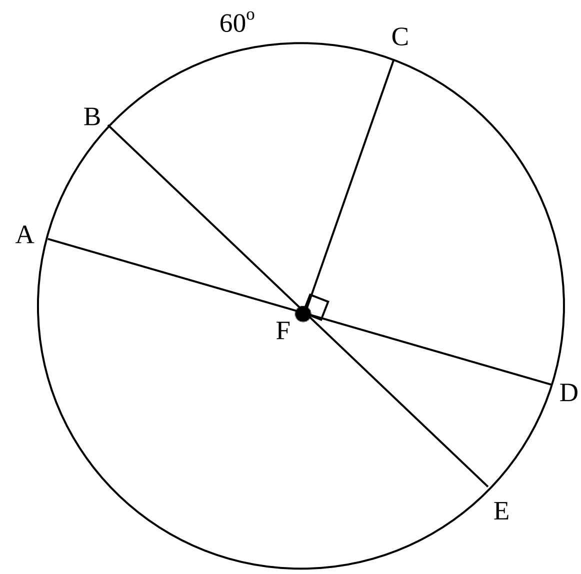
21. mMW 22. mRM 23. $mMTS$ 24. $mRMW$

25. $mSWT$

Lesson 1: Introduction to Circles Homework

Find the measure of each of the following

Given that $\overline{BE}$ *and* $\overline{AD}$ *are diameters,* $\overline{FC} \perp \overline{AD}$ *and* $mBC = 60$.



1. mCD

2. mAB

3. $m\angle BFC$

4. $m\angle AFB$

5. $m\angle DFE$

6. mDE

7. mAE

8. $mBAE$

9. $m\angle AFC$

10. $mEAC$

11. $m\angle CFE$

12. $mBCA$

13. $m\angle AFE$

Lesson 2: Central & Inscribed Angles

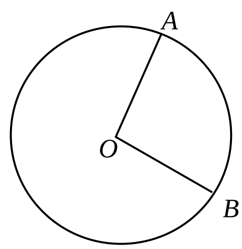
AIM: → ***To define central and inscribed angles***
→ ***To solve problems in circle geometry regarding central and inscribed angles***

All angle problems within circle geometry can essentially be classified into three subcategories: (1) angles within a circle, (2) angles on a circle, and (3) angles formed outside the circle. Today we will focus on central angles and inscribed angles; these angles appear within a circle and on a circle, respectively.

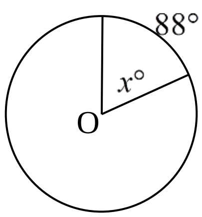
Central Angle

Any central angle formed (an angle with its vertex at the center of the circle) has a measure equal to the arc that it intercepts.

Ex. 1 In the following diagram of circle *O* it is given that $m\angle AOB = 112^\circ$. Find mAB.



1.



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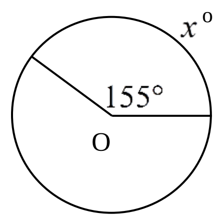
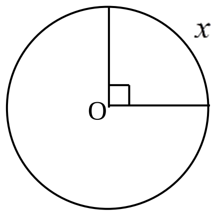


3.



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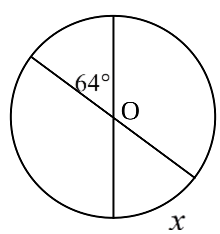
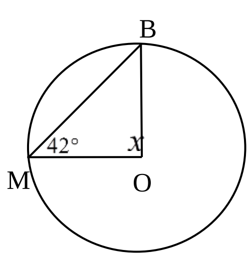
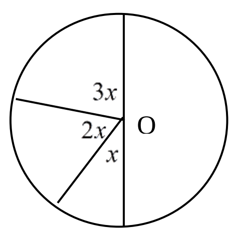


5.



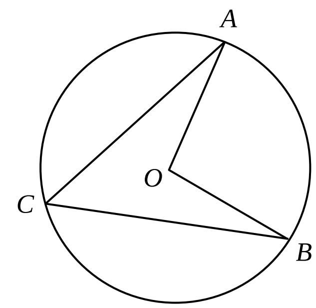
6.



3x
2x

Inscribed Angle An inscribed angle on a circle always measures one-half of its intercepted arc.

<u>Ex. 2</u> In the figure shown it is given that $mAB = 160^\circ$.

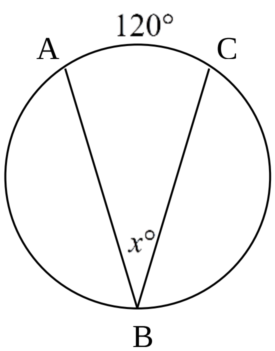


(a) Determine $m\angle AOB$.

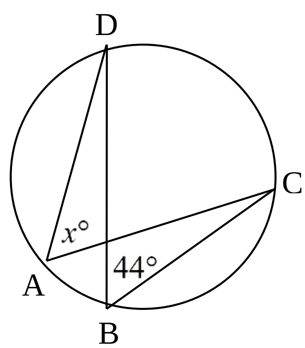
(b) Determine $m\angle ACB$.

Find x:

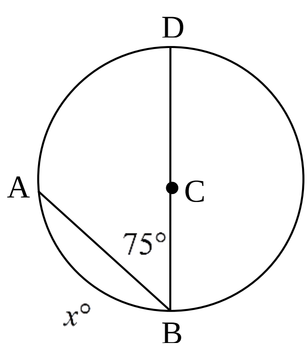
1.



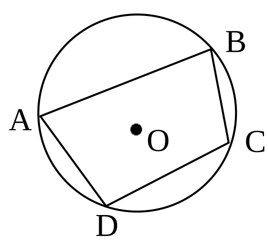
2.



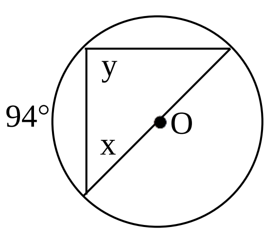
3.



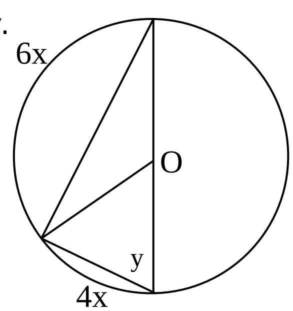
4. Quadrilateral ABCD is inscribed in circle O. If $\overset{\frown}{AB} = 140^\circ$ and $\overset{\frown}{BC} = 72^\circ$, find $m\angle ADC$.



5. Find *x* **and** *y*.

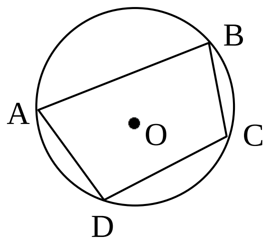


6. Find x **and** y.

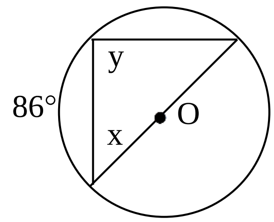


Lesson 2: Central & Inscribed Angles HOMEWORK

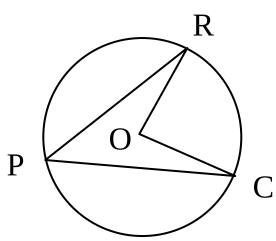
1. Quadrilateral ABCD is inscribed in circle O. If $\overset{\frown}{AB} = 125°$ and $\overset{\frown}{BC} = 83°$, find $m\angle ADC$.



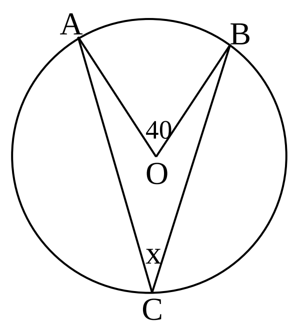
2. Find *x* **and** *y*.



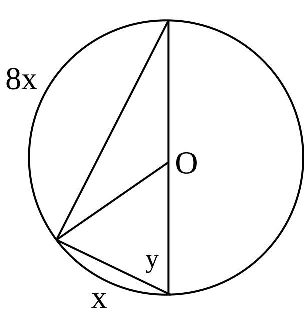
3. In circle O, $m\angle ROC = 88°$. Find $m\angle RPC$.



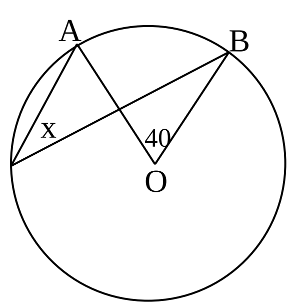
4. Find *x*.



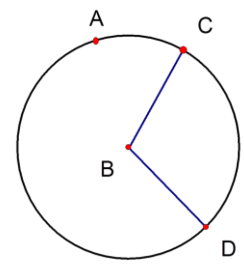
5. Find *x* **and** *y*.



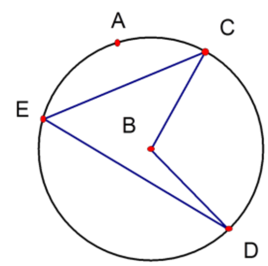
6. Find *x*.



7. In the following diagram, $m\angle CBD = 105^{\circ}$. Find $m\overset{\frown}{CD}$ and $m\overset{\frown}{CAD}$.



8. In the following diagram, $m\angle CBD = 120^{\circ}$. Find $m\angle CED$.

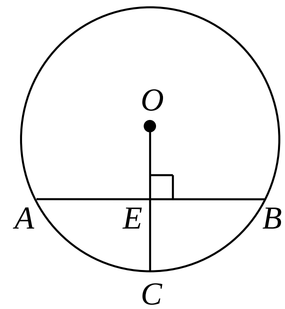


Lesson 3: Lengths of Segments – Chords

AIM: → *To define and apply theorems regarding perpendicular bisectors of chords*

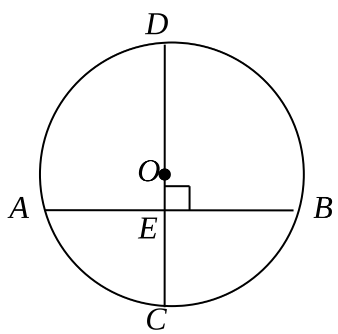
Thm: A radius drawn perpendicular to a chord bisects the chord.

(The converse of this theorem is also true: If a radius bisects a chord, then it is also perpendicular to it.)

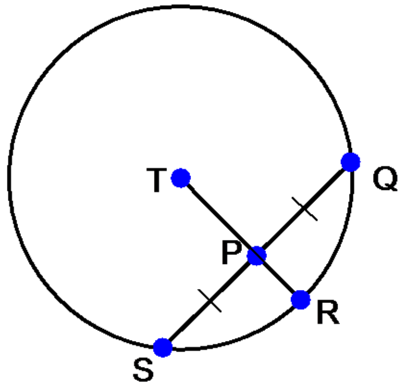


$\overline{AE} \cong \overline{BE}$

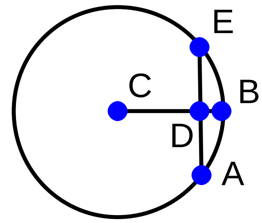
Ex. 1 In circle *O*, diameter $\overline{DOC}$ is perpendicular to chord $\overline{AB}$ at *E*, *AB* = 24, and *OE* = 5. Find the measure of the radius of the circle.



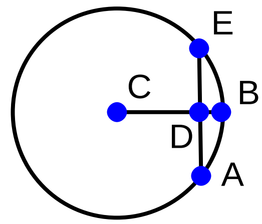
Ex. 2 a) If QS = 14, what is QP? b) What is $m\measuredangle TPQ$?



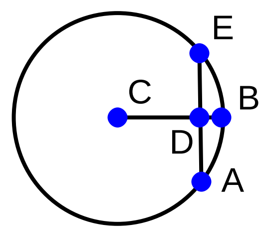
Ex. 3 AD=15, AC=17. Find the length of CD.



Ex. 4 AE=12, AC=10. Find the length of CD.

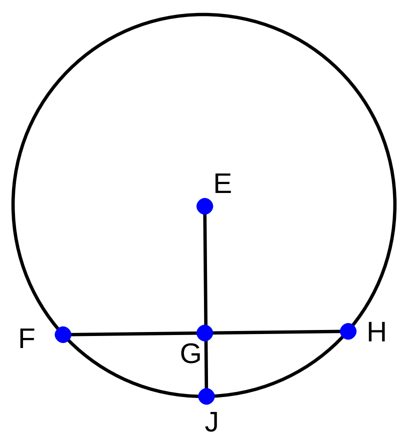


Ex. 5 AE=24, CD=9. Find the length of the radius of circle C.



Ex. 6

a. EG=24 and EF=25. Find the length of FH.

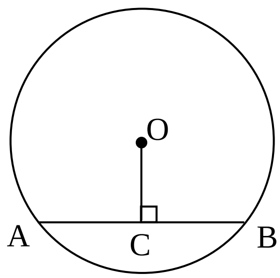


b. EG=15 and FH 40 Find the radius of circle E.

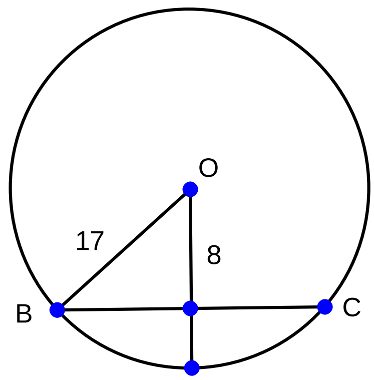
c. EF=2.7 and FH=3.2. Find the distance of EG. to the *nearest tenth*

Lesson 3: Lengths of Segments – Chords HOMEWORK

1. Find the length of the radius of circle O to the *nearest hundredth*, given $AB = 24$ and $OC = 12$.



2. Circle O has a radius of 17 units. Chord BC, with endpoints on circle O is 8 units from point O. Find BC.



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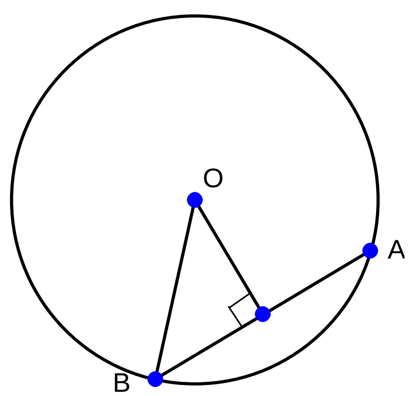
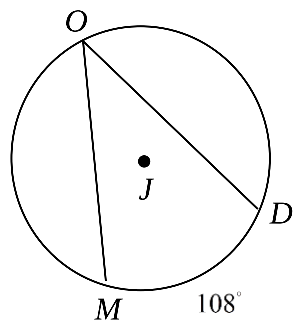
3. Find the distance from chord AB to the center of the circle, given that the radius is 5 and AB=6 .

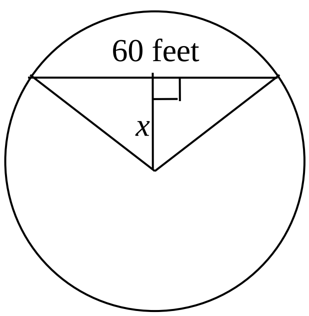


4. Given $\odot J$, find the measure of $\angle MOD$.



Challenge: I'll add 3 points to your next quiz if you can get this correct:

A 60-foot rope is attached from a point on a large circular pool to another point on the pool. If the circumference of the pool is 100π feet, find the number of feet (perpendicular distance) between the center of the pool and the rope, as illustrated in the diagram below.



Lesson 4: Lengths of Segments – Tangents

AIM: → To define and apply the theorem:
♦ Two tangents to a circle from the same external point are congruent.

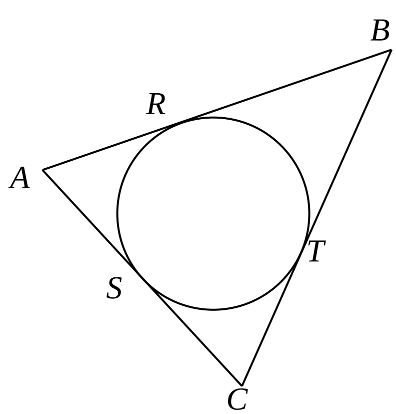
Thm: If two tangent segments are drawn to a circle from the same exterior point terminating at their respective points of tangency, then the tangent segments are congruent.

Picture:

In other words, if two tangents start at the same point and extend to the same circle, then they have *equal* lengths.

Definition: **INSCRIBED CIRCLE**: A circle that is tangent to every side of a polygon is *inscribed* in the polygon. (see exercise 1 for an example).

Ex 1. Using the diagram below and the given segment lengths, find the perimeter of ΔABC.

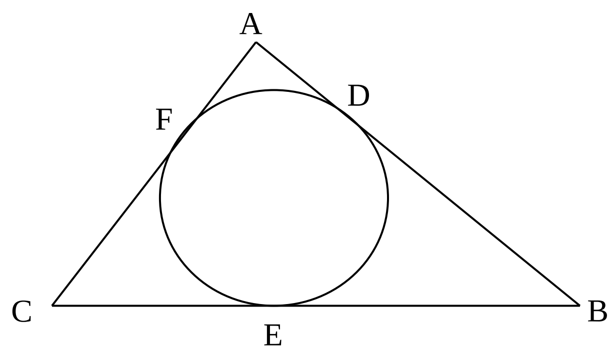


$AR = 5$

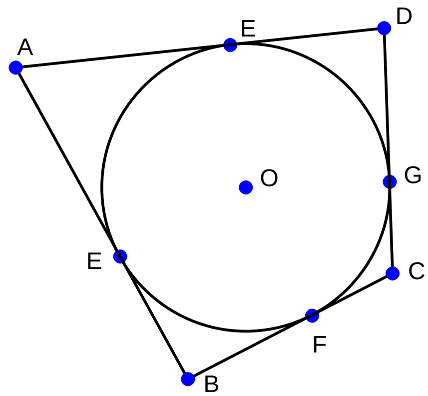
$BT = 8$

$CS = 6$

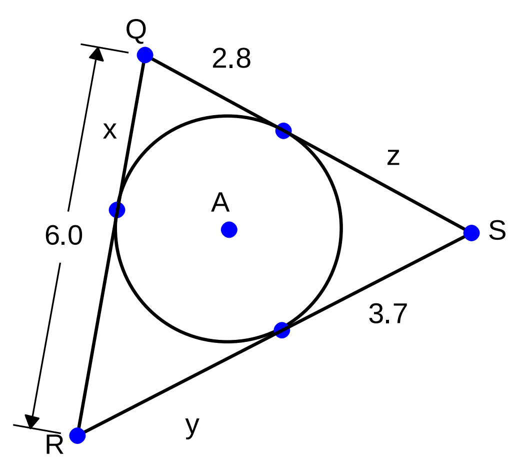
Ex.2. A circle is *inscribed* inside ΔABC such that points D, E, and F are points of tangency. Let $\overline{AD}=5$, $\overline{EB}=10$, and $\overline{CF}=7$. Find the lengths of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.



Ex 2. In the diagram, circle O is inscribed in quadrilateral ABCD and E, F, G and H are points of tangency of the sides. If AH=6, DG=4, CF=2 and BF=3, what is the perimeter of quadrilateral ABCD?

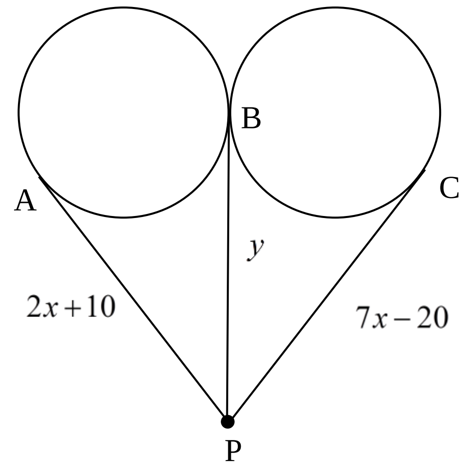


Ex. 4. Given the following figure, find x, y, z and the perimeter of triangle QRS.



Lesson 4: EXTENSIONS AND HOMEWORK:

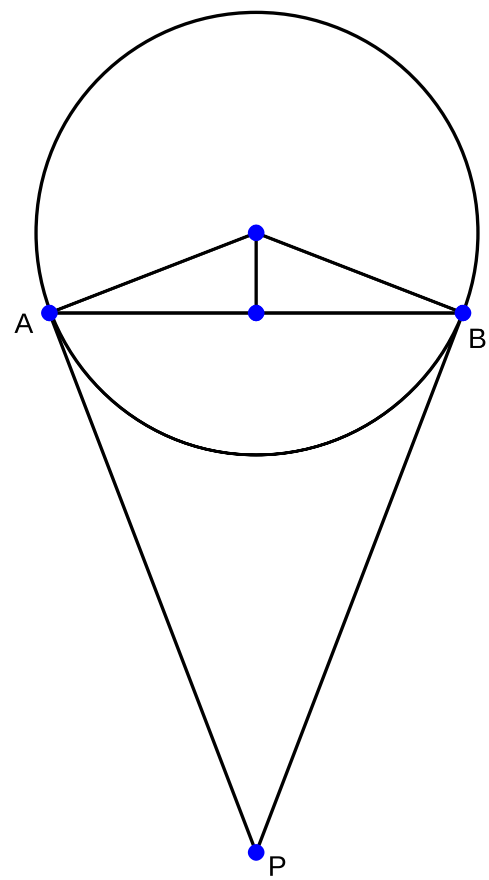
5. Find *x* and *y*, given that $\overline{PA}$, $\overline{PB}$, and $\overline{PC}$ are tangents.



Ex. 6 In the following diagram, segments PA and PB are tangent to the circle at A and B respectively.

Chord AB is 3 units from the center of the circle.

If the radius of the circle is 5 units and PA=12, find the perimeter of triangle PAB.



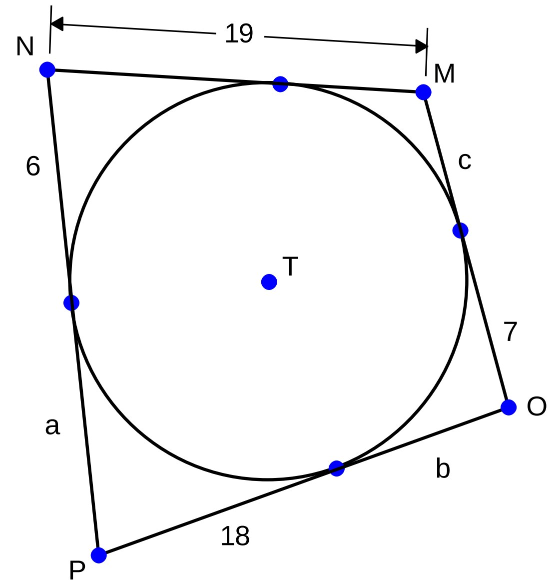
7. Circle O is inscribed in quadrilateral MNOP.

a. Solve for a.

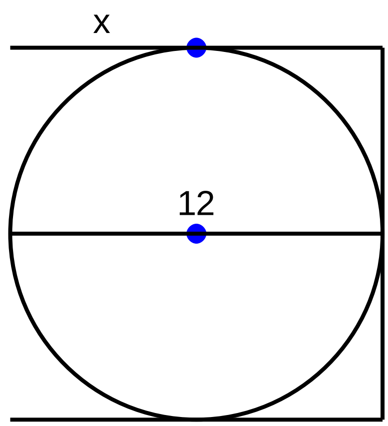
b. Solve for b.

c. Solve for c

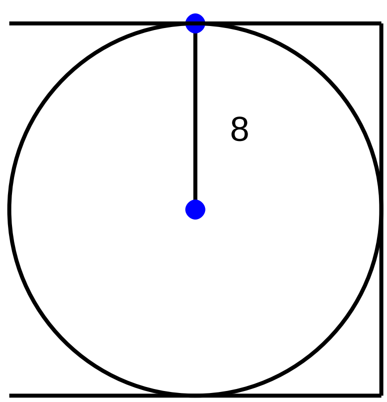
d. Find the perimeter of MNOP.



8. Find x AND the perimeter of the square.



9. Find the perimeter of the square.



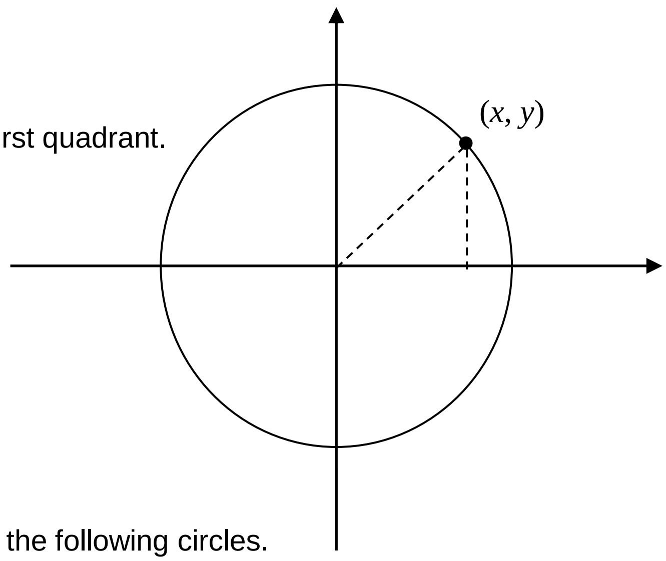
Lesson 5: Graphing Circles in the Coordinate Plane

AIM: → *To develop the equation of a circle* → *To graph circles in the coordinate plane*

Circle **: the collection of all points that are a fixed distance (the *radius*) from a fixed point (the *center*).**

Circles with Centers at the Origin (0,0)

Consider a point (x, y) located somewhere in the first quadrant.



$$x^2 + y^2 = r^2$$
(r is the radius)

Ex. 1 Identify the center and the radius of each of the following circles.

(a) $x^2 + y^2 = 16$ (b) $x^2 + y^2 = 25$ (c) $x^2 + y^2 = 9$ (d) $x^2 + y^2 = 1$

Graphing Circles from their Equations

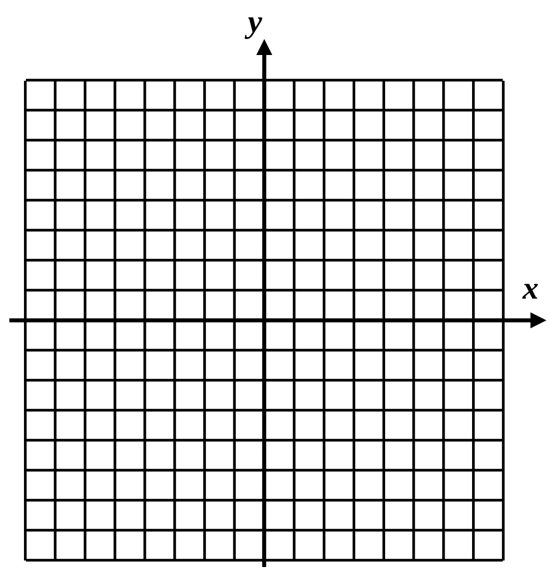
Graph the circle $x^2 + y^2 = 25$.

1. Identify the center and the radius of the circle:

Center:

Radius:

2. Draw four representative points.



Ex. 2 On the same set of axes, draw the circle whose equation is $x^2 + y^2 = 16$.

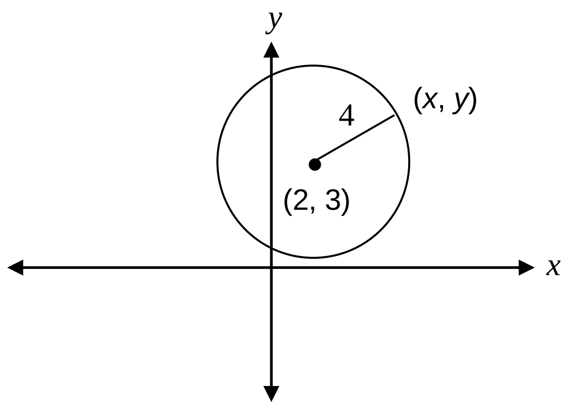
✦How do we find the equation of a circle if it is not center at the origin? How do we graph it?✦

In order to write an equation for a given circle not centered at the origin, we make use of the **distance formula:**

➡ In a coordinate plane, the distance, *d*, from (x_1, y_1) to (x_2, y_2) is given by the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

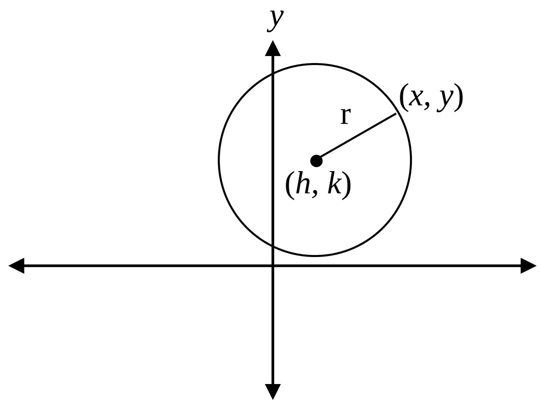
Ex. 1 Write an equation for the circle with center (2, 3) and radius 4.



Solution: Every point (*x*, *y*) on this circle is at a distance of 4 units from (2, 3). In the space below, use this fact together with the distance formula to write an equation for this circle:

Now, square both sides to eliminate the radical. We will then have the equation expressed in **center-radius form** (or **standard form**):

Write an equation for the circle with center (h, k) and radius r.



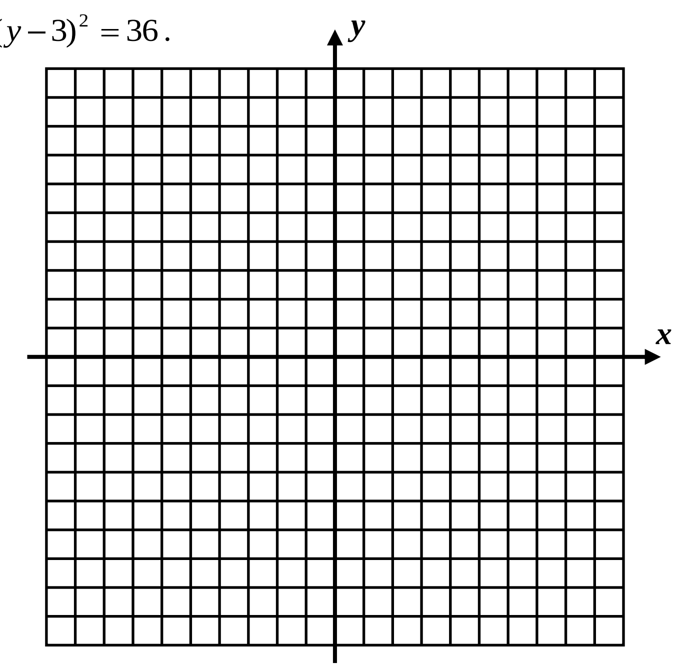
Ex. 2 Write, in center-radius form, an equation for the circle with the given center, C, and the given radius, r.

(a) $C(0,0)$; $r = 5$ (b) $C(4, -5)$; $r = 3$ (c) $C(-4, 0)$; $r = \sqrt{10}$

Ex. 3 Graph the circle whose equation is $(x-1)^2 + (y-3)^2 = 36$.

Center:

Radius:



Lesson 5: Graphing Circles in the Coordinate Plane MORE PRACTICE

1. Circle *O* has equation $(x+4)^2+(y-1)^2=81$.

   (a) What is the radius of circle *O*?

   (b) What are the coordinates of the center of circle *O*?

2. Graph the circle whose equation is $x^2+(y+3)^2=16$.

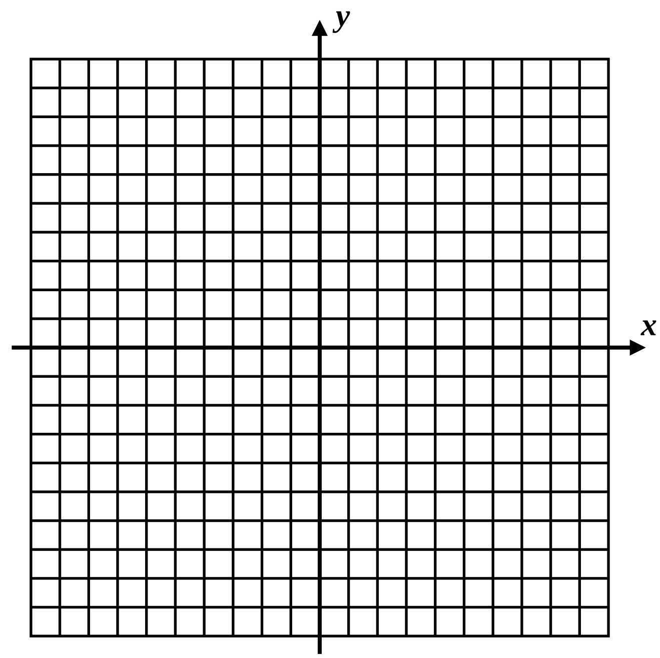
   Center:

   Radius:

   On the same set of axes, graph the circle whose equation is $(x-2)^2+(y-3)^2=4$.

   Center:

   Radius:



3. What is the equation of a circle with center (–3,1) and radius 7? 3. __________

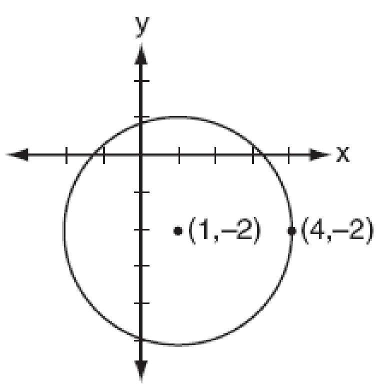
   (1) $(x-3)^2+(y+1)^2=7$

   (2) $(x-3)^2+(y+1)^2=49$

   (3) $(x+3)^2+(y-1)^2=7$

   (4) $(x+3)^2+(y-1)^2=49$

4. Which equation represents the circle shown in the accompanying graph? 4. __________



   (1) $(x-1)^2-(y+2)^2=9$

   (2) $(x-1)^2+(y+2)^2=9$

   (3) $(x+1)^2-(y-2)^2=9$

   (4) $(x+1)^2+(y-2)^2=9$

Lesson 5: Graphing Circles in the Coordinate Plane MORE PRACTICE AND HOMEWORK

In exercises 1 – 8, identify the center and the radius of each circle given its equation.

1. $x^2 + y^2 = 100$

2. $x^2 + y^2 = 1$

3. $(x-5)^2 + (y-1)^2 = 9$

4. $(x+3)^2 + (y-7)^2 = 25$

5. $(x+2)^2 + (y+4)^2 = 64$

6. $(x-1)^2 + (y+4)^2 = 36$

7. $x^2 + y^2 = 121$

8. $x^2 + (y-5)^2 = 49$

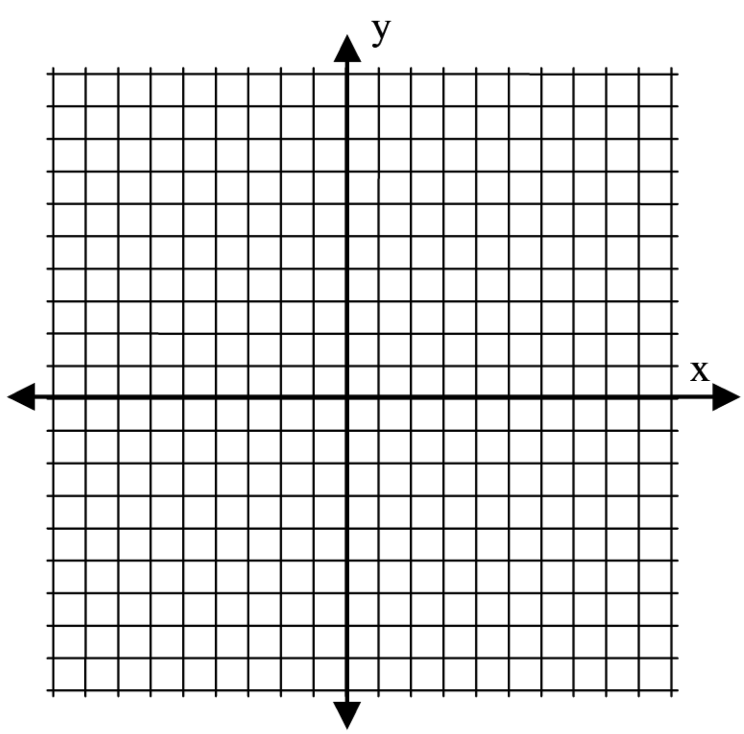
In exercises 9 – 12, graph the given equation on the axes to the right. Label each graph with the problem number.

9. $(x-3)^2 + (y-4)^2 = 9$

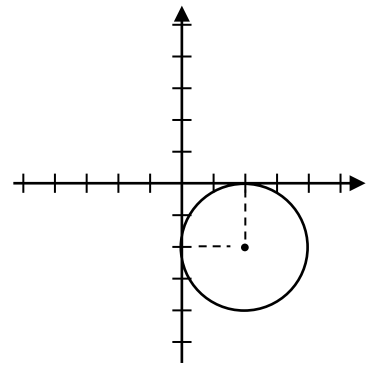
10. $x^2 + y^2 = 36$

11. $(x+4)^2 + (y-2)^2 = 25$

12. $(x-4)^2 + (y+2)^2 = 4$



13. Which of the following equations could represent the graph shown at the right?



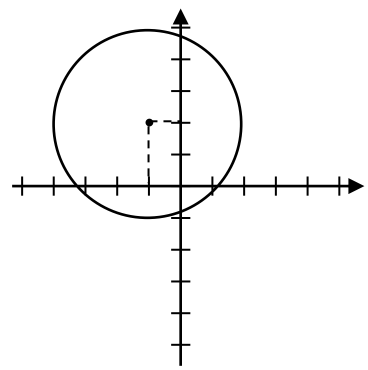
(a) $(x-2)^2+(y+2)^2=2$

(b) $(x-2)^2+(y+2)^2=4$

(c) $(x+2)^2+(y-2)^2=2$

(d) $(x+2)^2+(y-2)^2=4$

14. Which of the following equations could represent the graph shown at the right?



(a) $(x+1)^2+(y-2)^2=36$

(b) $(x-1)^2+(y+2)^2=9$

(c) $(x+1)^2+(y-2)^2=9$

(d) $(x-1)^2+(y-2)^2=36$

15. A sunflower has a circular shape that can be described by the equation $(x+7)^2+(y-2)^2=22$. Where is the center of the sunflower located?

(a) (7, -2) (c) (-7, -2)

(b) (-7, 2) (d) (7, 2)

16. A gardener wishes to design a circular flower-bed that has a diameter of 10 feet and a center at the point (5, 8). Which of the following equations would represent his garden?

(a) $x^2+y^2=100$ (c) $(x-5)^2+(y-8)^2=25$

(b) $x^2+y^2=25$ (d) $(x+5)^2+(y+8)^2=25$

17. A circle has an equation given by $x^2+y^2=20$.

(a) Find the radius of the circle in simplest radical form.

(b) Find the circumference of the circle to the *nearest tenth*.